Training in Advanced Low Energy Nuclear Theory

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Introduction

1. General features of inelastic scattering
   - Formal treatment: Coupled-Channels method
   - Partial wave analysis
   - Scattering wavefunction
   - Coupling potentials in the channel basis
2. Models for inelastic scattering
   - Coulomb excitation
   - Collective nuclear excitations: rotor model
   - Microscopic folding models
3. Input examples for inelastic scattering
   - Collective models
   - Example for rotor model: ^{7}Li + ^{208}Pb
   - Example for collective model with general matrix elements: ^{16}O+ ^{208}Pb

Remainder: direct versus compound reactions

DIRECT: elastic, inelastic, transfer,...

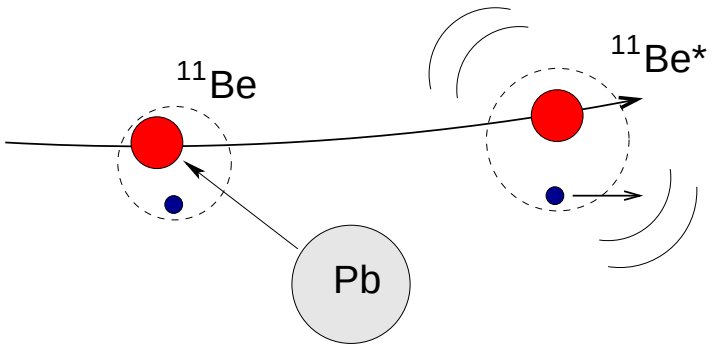
- only a few modes (degrees of freedom) involved
- small momentum transfer
- angular distribution asymmetric about $\pi/2$ (peaked forward)

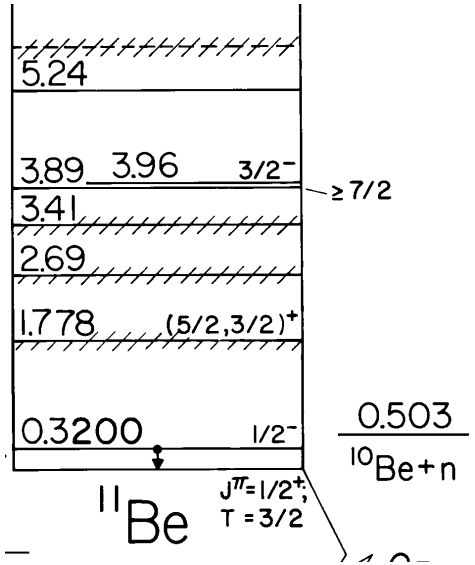
COMPOUND: complete, incomplete fusion.

- many degrees of freedom involved
- large amount of momentum transfer
- "lose of memory" $\Rightarrow$ almost symmetric distributions forward/backward

Inelastic scattering to bound states

- Nuclei are not inert or *frozen* objects; they do have an internal structure of protons and neutrons that can be modified (excited) during the collision.
- Quantum systems exhibit, in general, an energy spectrum with bound and unbound levels.



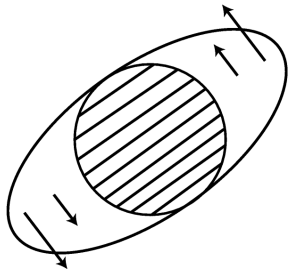


Inelastic scattering

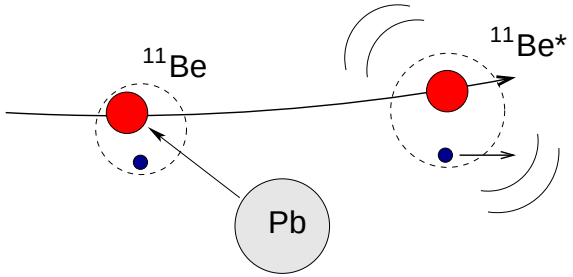
- Direct reactions → nuclei make "glancing" contact and separate immediately.
- Energy/momentum transferred from relative motion to internal motion so the projectile and/or target are left in an excited state.
- Involve small number of degrees of freedom.
- The colliding nuclei preserve their identity: $a + A \rightarrow a^* + A^*$
- Typically, they are peripheral (surface) processes.

Models for inelastic excitations

1. COLLECTIVE: Involve a collective motion of several nucleons which can be interpreted macroscopically as rotations or surface vibrations of the nucleus.

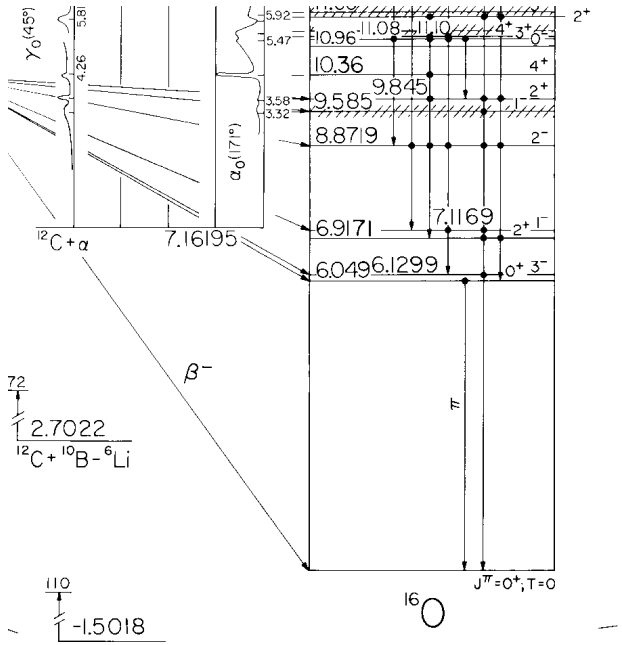


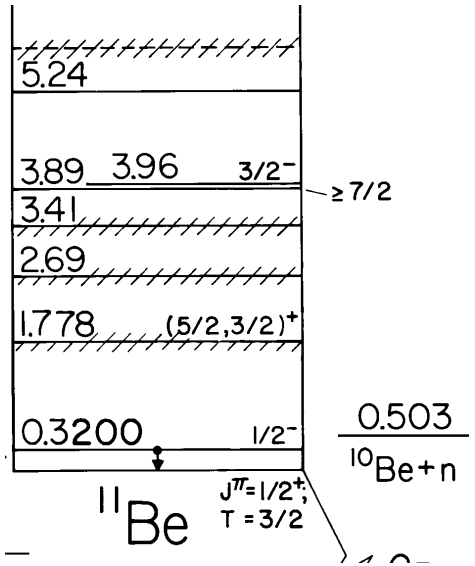
2. SIGLE-PARTICLE: Involve the excitation of a nucleon or cluster.



Models for inelastic excitations

Microscopically, what we describe in both cases are quantum transitions between discrete or continuum states:





☞*Collective excitations can be regarded as a coherent superposition of many single-particle excitations.*

Energy balance for inelastic scattering

- For projectile excitation: $a + A \rightarrow a^* + A$

$$E^i_{\rm cm} + M_a c^2 + M_A c^2 = E^f_{\rm cm} + M^*_a c^2 + M_A c^2$$

$M_{a^*} = M_a + E_x$ (E_x=excitation energy)

- Q-value:

$$Q = M_a c^2 + M_A c^2 - M^*_a c^2 - M^2_A c^2 = -E_x < 0$$

$$E^f_{\rm cm} = E^i_{\rm cm} + Q$$

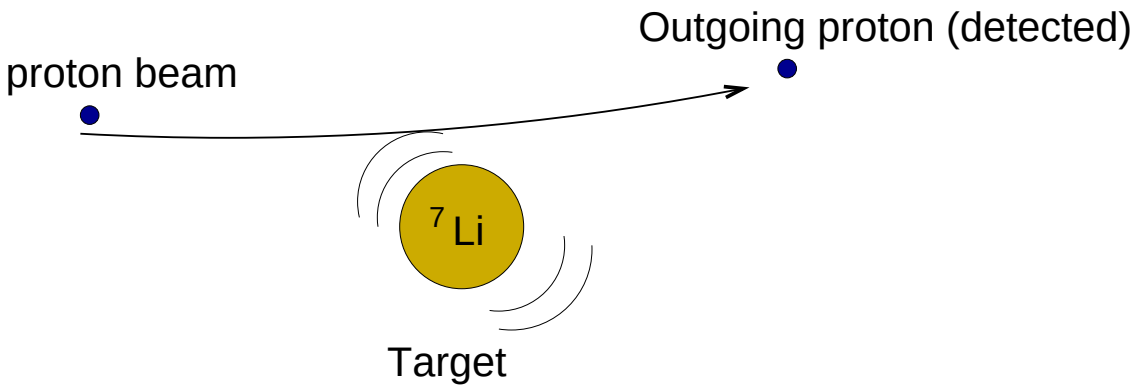
- So

$$E_x = E^i_{cm} - E^f_{cm}$$

What do we measure in an inelastic scattering experiment?

☞ In general, one measures the scattering angle and energy of outgoing particles.

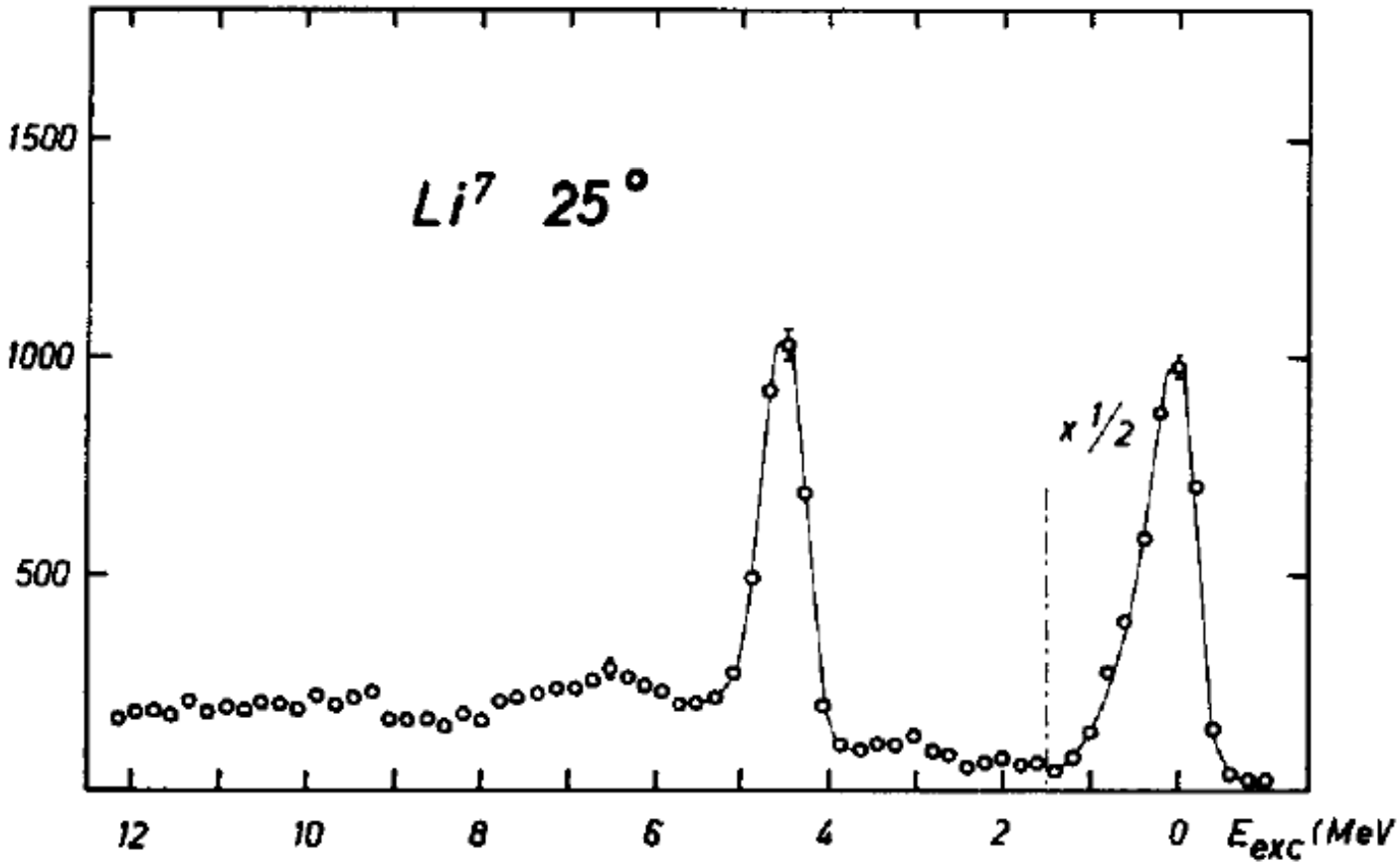
EXAMPLE: $p+^{7}Li \rightarrow p+^{7}Li^{*}$



☞ *Eg. energy and angular distribution of the outgoing protons.*

What do we measure in an inelastic scattering experiment?

☞ The proton energy carries information on the ^{7}Li excitation spectrum.



Data from Nuclear Physics 69 (1965) 81-102

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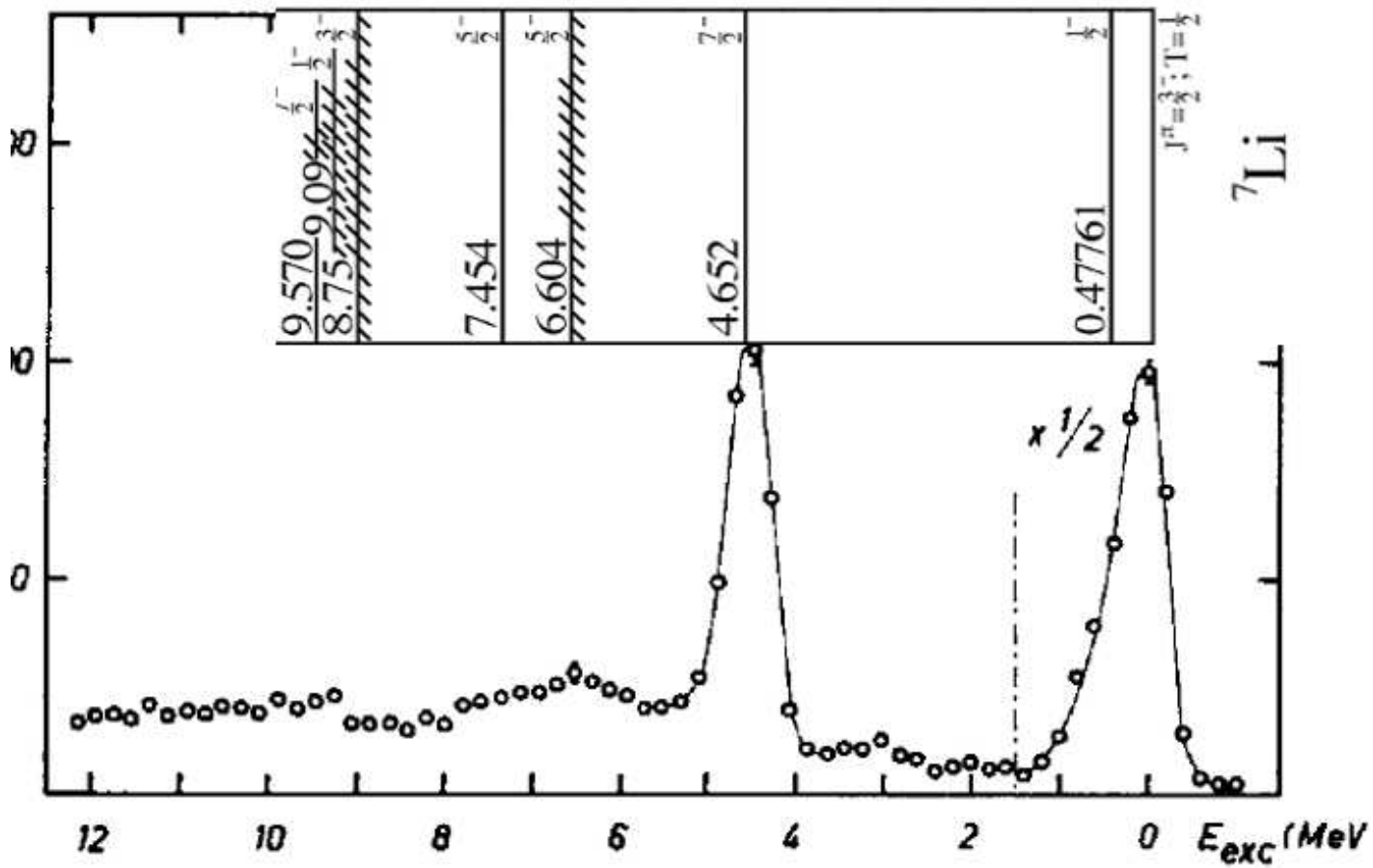
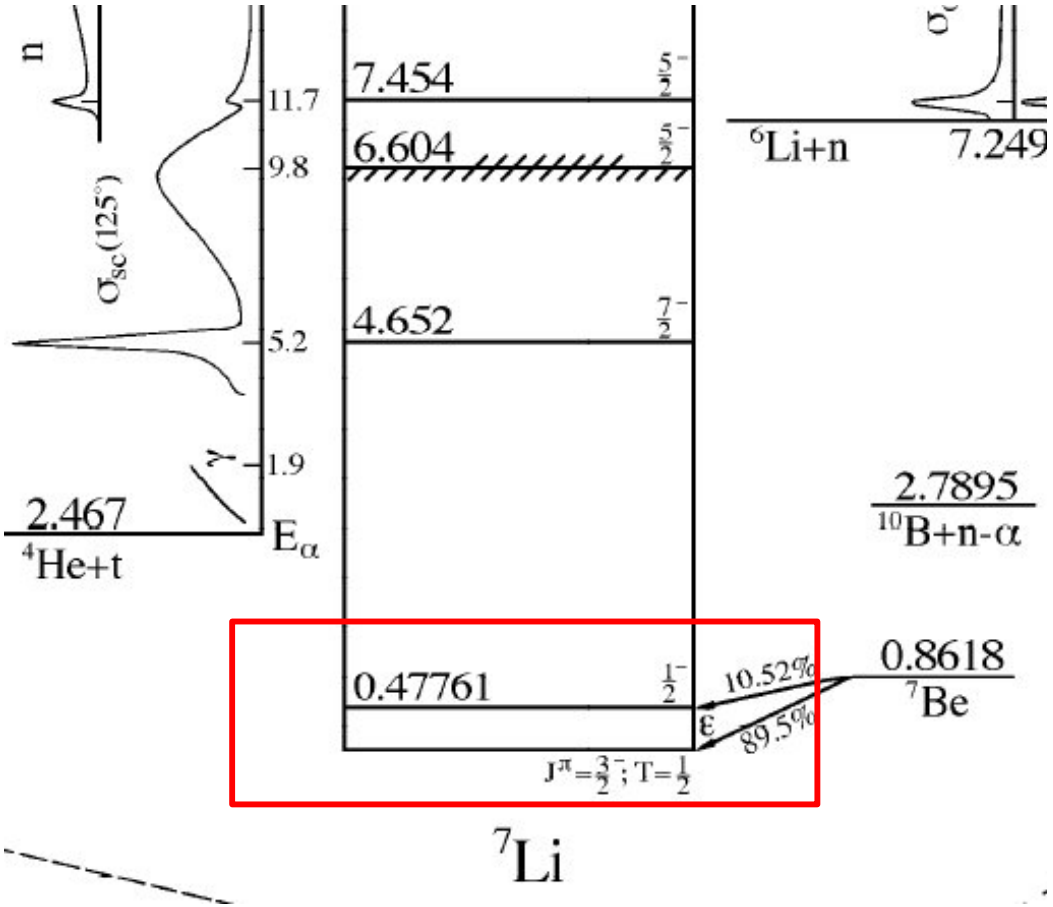


What information do we get from an inelastic scattering experiment?

- The proton energy spectrum shows peaks which correspond to the states of the target (^{7}Li)
- The heights of peak ($\sim$ cross section) are different for each state $\Rightarrow$ not all states are populated with the same probability.
- Some peaks are narrow, other are broad. Why?...
- Above a certain excitation energy, the spectrum becomes continuous and structureless.

What information do we get from an inelastic scattering experiment?



Formal treatment: the coupled-channels method

We need to incorporate explicitly in the Hamiltonian the internal structure of the nucleus being excited (eg. target).

$$H = T_R + h(\xi) + V(\mathbf{R}, \xi)$$

- T_R: Kinetic energy for projectile-target relative motion.
- $\{\xi\}$: Internal degrees of freedom of the target (depend on the model).
- $h(\xi)$: Internal Hamiltonian of the target.

$$h(\xi)\phi_n(\xi) = \varepsilon_n \phi_n(\xi)$$

- $V(\mathbf{R}, \xi)$: Projectile-target interaction, eg:

$$V(\mathbf{R}, \xi) = \sum_{i=1}^{N} V_{pi}(\mathbf{r}_{pi})$$

Model wavefunction (target excitation)

We expand the total wave function in a subset of internal states (the P space):

$$\Psi_{\rm model}(\mathbf{R},\xi) = \phi_0(\xi)\chi_0(\mathbf{R}) + \sum_{n>0} \phi_n(\xi)\chi_n(\mathbf{R})$$

- $\phi_0(\xi)$: target ground state wave function.
- $\chi_0(\mathbf{R})$: describes the projectile-target relative motion with the target in the initial state $\Rightarrow$ elastic scattering

- $\phi_n(\xi)$: target internal wave function in the excited state n.
- $\chi_n(\mathbf{R})$: describes the projectile-target relative motion with the target in the excited state $n \Rightarrow$ inelastic scattering

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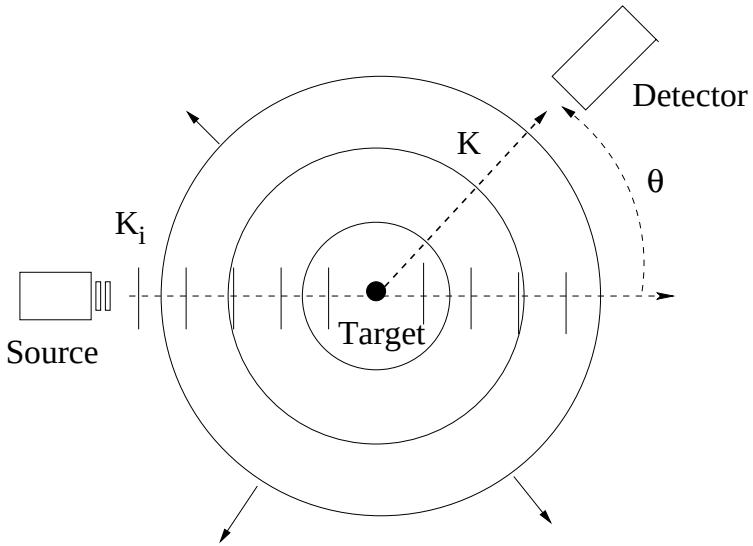
$\chi_n(\mathbf{R}) \rightarrow$ unknowns

Physical meaning of $\chi_n(\mathbf{R})$: scattering observables

☞*The quantum scattering is a wave process:*



- The incident projectile is described by a plane wave $\rightarrow e^{i\mathbf{K}_i\cdot\mathbf{R}}$
- The scattered projectile is described at large distances by outgoing spherical waves: $\rightarrow \frac{e^{iK_fR}}{R}$ (more generally, $H^{(+)}(K_f, R)$)

Physical meaning of $\chi_n(\mathbf{R})$: scattering observables

In the problem WITHOUT internal degrees of freedom...

- Scattering amplitude: $f(\theta)$

$$\Psi^{(+)}_{\mathbf{K}_i}(\mathbf{R}) = e^{i\mathbf{K}_i\cdot\mathbf{R}} + \chi^{(+)}_{\mathbf{K}_i}(\mathbf{R}) \xrightarrow{R\gg} e^{i\mathbf{K}_i\cdot\mathbf{R}} + f(\theta)\frac{e^{iK_iR}}{R}$$

- Cross section:

$$\boxed{\frac{d\sigma}{d\Omega}(\theta) = |f(\theta)|^2}$$

☞ *$f(\theta)$ is the coefficient of the outgoing spherical wave at large distances.*

☞ *The square of $f(\theta)$ gives the probability that the particle be scattered at an angle θ.*

Physical meaning of $\chi_n(\mathbf{R})$: scattering observables

In the problem WITH internal degrees of freedom...

- Scattering amplitude:

$$\Psi^{(+)}_{\mathbf{K}_i}(\mathbf{R},\xi) \rightarrow \left\{ e^{i\mathbf{K}_0\cdot\mathbf{R}} + f_{0,0}(\theta)\frac{e^{iK_0R}}{R} \right\}\phi_0(\xi) \qquad \text{(elastic)}$$

$$\Psi^{(+)}_{\mathbf{K}_i}(\mathbf{R},\xi) \rightarrow f_{n,0}(\theta)\frac{e^{iK_nR}}{R}\phi_n(\xi), \qquad n \neq 0 \quad \text{(non – elastic)}$$

- Cross sections:

$$\frac{d\sigma_n(\theta)}{d\Omega} = \frac{K_n}{K_0}|f_{n,0}(\theta)|^2 \equiv |\tilde{f}_{n,0}(\theta)|^2$$

☞ *$f_{n,0}(\theta)$ is the coefficient of the outgoing spherical wave $\chi^{(+)}_n(\mathbf{R})$ at large distances.*

☞ *The square of $\tilde{f}_{n,0}(\theta)$ gives the probability that the particle be scattered at an angle θ with the target ending up in the excited state n.*

Calculation of $\chi_n^{(+)}(\mathbf{R})$: the coupled equations

- The model wavefunction must satisfy the Schrödinger equation:

$$[H - E]\Psi^{(+)}_{\rm model}(\mathbf{R}, \xi) = 0$$

- Projecting onto the internal states one gets a system of coupled-equations for the functions $\{\chi_n(\mathbf{R})\}$:

$$\boxed{[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

- Coupling potentials:

$$\boxed{V_{n,n'}(\mathbf{R}) = \int d\xi \phi_{n'}(\xi)^* V(\mathbf{R}, \xi)\phi_n(\xi)}$$

☞ *$\phi_n(\xi)$ will depend on the structure model (collective, single-particle,etc).*

Optical Model vs. Coupled-Channels method

Optical Model

- The Hamiltonian:
  $H = T_R + V(\mathbf{R})$
- Internal states:
  Not considered explicitly
- Model wavefunction:
  $\Psi_{\rm el}(\mathbf{R}) \equiv \chi_0(\mathbf{R})$
- Schrödinger equation:
  $[H - E]\chi_0(\mathbf{R}) = 0$

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Coupled-channels method

- The Hamiltonian:
  $H = T_R + h(\xi) + V(\mathbf{R}, \xi)$
- Internal states:
  $h(\xi)\phi_n(\xi) = \varepsilon_n \phi_n(\xi)$
- Model wavefunction:
  $\Psi_{\rm model}(\mathbf{R}, \xi) = \phi_0(\xi)\chi_0(\mathbf{R}) + \sum_{n>0} \phi_n(\xi)\chi_n(\mathbf{R})$
- Schrödinger equation:

$$[H - E]\Psi_{\rm model}(\mathbf{R}, \xi) = 0$$

$$\Downarrow$$

$$[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})$$

DWBA approximation as 1st order CC

- Two-states model $n = 0, 1$:

$$\Psi(\mathbf{R}, \xi) = \underbrace{\phi_0(\xi)\chi_0(\mathbf{R})}_{\text{elastic}} + \underbrace{\phi_1(\xi)\chi_1(\mathbf{R})}_{\text{inelastic}}$$

- Coupled-channels equations:

$$[E - \varepsilon_0 - T_0 - V_{00}(\mathbf{R})]\chi_0(\mathbf{R}) = V_{01}(\mathbf{R})\chi_1(\mathbf{R})$$
$$[E - \varepsilon_1 - T_1 - V_{11}(\mathbf{R})]\chi_1(\mathbf{R}) = V_{10}(\mathbf{R})\chi_0(\mathbf{R})$$

- Iterative solution of the CC equations (DWBA):

$$[E - \varepsilon_0 - T_0 - V_{00}(\mathbf{R})]\chi_0(\mathbf{R}) \approx 0$$
$$[E - \varepsilon_1 - T_1 - V_{11}(\mathbf{R})]\chi_1(\mathbf{R}) \approx V_{10}(\mathbf{R})\chi_0(\mathbf{R})$$

DWBA approximation as 1st order CC

- Asymptotically:

$$\chi_1^{(+)}(\mathbf{R}) \to f_{10}(\theta)\frac{e^{iK_1R}}{R}$$

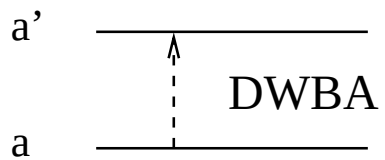
with (not proven here!)

$$f_{10}(\theta) = -\frac{2\mu}{4\pi\hbar^2}\int d\mathbf{R}\widetilde{\chi}_1^{(-)}(\mathbf{K}_1,\mathbf{R})V_{10}(\mathbf{R})\widetilde{\chi}_0^{(+)}(\mathbf{K}_0,\mathbf{R})$$

where $\widetilde{\chi}_0(\mathbf{K}_0,\mathbf{R}), \widetilde{\chi}_1(\mathbf{K}_1,\mathbf{R})$ are solutions of:

$$[E-\varepsilon_0-T_0-V_{00}(\mathbf{R})]\widetilde{\chi}_0(\mathbf{K}_0,\mathbf{R}) = 0$$
$$[E-\varepsilon_1-T_1-V_{11}(\mathbf{R})]\widetilde{\chi}_1(\mathbf{K}_1,\mathbf{R}) = 0$$

☞ *The DWBA approximation amounts at solving the CC equations to 1st order (Born approximation)*



Summary of the DWBA approximation for inelastic scattering

- Assume that $V(\mathbf{R},\xi) = V_0(\mathbf{R}) + \Delta(\mathbf{R},\xi)$
- Calculate the "unperturbed" initial and final waves:

$$[E - \varepsilon_i - T_i - V_{0,i}(\mathbf{R})]\widetilde{\chi}_i^{(+)}(\mathbf{K}_i,\mathbf{R}) = 0$$
$$\left[E - \varepsilon_f - T_f - V_{0,f}(\mathbf{R})\right]\widetilde{\chi}_f^{(+)}(\mathbf{K}_f,\mathbf{R}) = 0$$

- Treat $\Delta(\mathbf{R},\xi)$ in first order (Born approximation):

$$f_{fi}(\theta) = -\frac{2\mu}{4\pi\hbar^2}\int d\mathbf{R}\,\widetilde{\chi}_f^{(-)}(\mathbf{K}_f,\mathbf{R})\Delta_{if}(\mathbf{R})\widetilde{\chi}_i^{(+)}(\mathbf{K}_i,\mathbf{R})$$

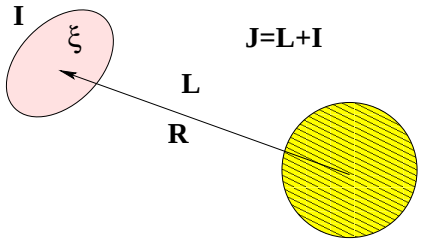
- $\Delta_{if}(\mathbf{R})$ is the transition potential

$$\Delta_{if}(\mathbf{R}) = \int d\xi\phi_f^*(\xi)\Delta(\xi,\mathbf{R})\phi_i(\xi)$$

Partial wave analysis: the channel basis

- The channel basis (projectile excitation):

$$\Phi^{JM_J}_{nLI}(\hat{R},\xi) = \sum_{M_I M_L} i^L Y_{LM_L}(\hat{R})\phi_{nIM_I}(\xi)\langle LM_L IM_I|JM_T\rangle$$
$$= \left[i^L Y_L(\hat{R}) \otimes \phi_{nI}(\xi)\right]_{JM_T}$$



- The total WF is expanded in the channel basis:

$$\Psi^{(+)}(\mathbf{R},\xi) = \sum_{\beta,J,M_T} C^{\beta,JM_T} \frac{\chi^J_\beta(R)}{R} \Phi^{JM_T}_\beta(\hat{R},\xi); \qquad \beta \equiv \{nLI\}$$

- For a total J, there will be a certain number (N) of channels β

Calculation of $\chi^J_\beta(R)$: radial coupled equations

- The coupled equations:

$$\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+\frac{\hbar^2 L(L+1)}{2\mu R^2}+\epsilon_n-E\right)\chi^J_\beta(R)+\sum_{\beta'}V^J_{\beta,\beta'}(R)\chi^J_{\beta'}(R)=0$$

- Coupling potentials in the channel-basis:

$$V^J_{\beta,\beta'}(R)=\int d\hat{R}\, d\xi \Phi^{JM_T}_\beta(\hat{R},\xi)^* V(\mathbf{R},\xi)\Phi^{JM_T}_{\beta'}(\hat{R},\xi)$$

Solution of the coupled equations: boundary conditions

- For each $J \Rightarrow N$ coupled diff. equations (one for each β) $\Rightarrow N$ indep. solutions
- Standard choice: build solutions characterized by a given entrance channel $\beta_i = \{n_i, L_i, I_i\}$:

$$\Psi^{(+)}_{\beta_i, JM_T}(\mathbf{R}, \xi) = \sum_{\beta} \frac{\chi^J_{\beta,\beta_i}(R)}{R} \Phi^{JM_T}_{\beta}(\hat{R}, \xi)$$

- The radial functions $\chi^J_{\beta;\beta_i}(R)$ verify:
  - ➪ Regular at the origin: $\chi^J_{\beta;\beta_i}(R = 0) = 0$
  - ➪ Asymptotic behaviour:

$$\chi^J_{\beta:\beta_i}(K_\beta, R) \rightarrow \frac{i}{2}\left[H^{(-)}_L(K_\beta R)\delta_{\beta,\beta_i} - S^J_{\beta,\beta_i} H^{(+)}_L(K_\beta R)\right]$$
$$\rightarrow \left[F_L(K_\beta R)\delta_{\beta,\beta_i} + T^J_{\beta,\beta_i} H^{(+)}_L(K_\beta R)\right]$$

$$\boxed{S^J_{\beta,\beta_i} = \delta_{\beta,\beta_i} + 2iT^J_{\beta,\beta_i}} \qquad \text{(Multi-channel S/T-matrix)}$$

Scattering wavefunction

- Wavefunction corresponding to the experimental condition:

$$\Psi^{(+)}_{\mathbf{K}_i,n_iI_iM_i}(\mathbf{R},\xi) = e^{i\mathbf{K}_i\cdot\mathbf{R}}\phi_{n_iI_iM_i}(\xi) + \chi^{(+)}_{\mathbf{K}_i,n_iI_iM_i}(\mathbf{R},\xi)$$

- Can be constructed making a l.c. of $\{\beta_i, J, M_T\}$ solutions:

$$\Psi^{(+)}_{\mathbf{K}_i,n_iI_iM_i}(\mathbf{R},\xi) = \sum_{\beta_i JM_T} C^{\beta_i,JM_T}_{I_iM_i}\Psi^{(+)}_{\beta_i,JM_T} = \sum_{\beta_i JM_T} C^{\beta_i,JM_T}_{I_iM_i}\sum_{\beta}\frac{\chi^J_{\beta,\beta_i}(R)}{R}\Phi^{JM_T}_{\beta}(\hat{R},\xi)$$

Comparing with the partial-wave expansion of $e^{i\mathbf{K}_i\cdot\mathbf{R}}$ [EXERCISE]:

$$C^{\beta_i,JM_T}_{I_iM_i} = \sum_{N_i}\frac{4\pi}{K_i}\langle L_iN_iI_iM_i|JM_T\rangle Y^*_{L_iN_i}(\hat{K}_i)$$

Scattering amplitude and cross sections

- The scattering amplitude is the coefficient of e^{iKR}/R:

$$\tilde{f}(\mathbf{K}_i,\mathbf{K})_{n_iI_iM_i;nIM} = \frac{2\pi i}{\sqrt{KK_i}} \sum_{JM_TL_iN_i} Y^*_{L_iN_i}(\hat{K}_i)\langle L_iN_iI_iM_i|JM_T\rangle$$
$$\times \sum_{nILN} \left(\delta_{n,n_i}\delta_{I,I_i}\delta_{L,L_i} - S^J_{nIL;n_iI_iL_i}\right)\langle LNIM|JM_T\rangle Y_{LN}(\hat{K})$$

- Elastic and inelastic cross sections:

$$\left(\frac{d\sigma}{d\Omega}\right)_{i\to n} = \frac{1}{2I_i+1}\sum_{M,M_i} |\tilde{f}(\mathbf{K}_i,\mathbf{K})_{n_iI_iM_i;nIM}|^2$$

Multipolar expansion of coupling potentials in $|IM\rangle$ basis

- General multipole expansion of $V(\mathbf{R},\xi)$:

$$V(\mathbf{R},\xi) = \sqrt{4\pi}\sum_{\lambda,\mu} V_{\lambda\mu}(R,\xi)Y_{\lambda\mu}(\hat{R})$$

$$V_{n,n'}(\mathbf{R}) \equiv \langle I_f M_f|V(\mathbf{R},\xi)|I_i M_i\rangle = \sqrt{4\pi}\sum_{\lambda,\mu}\langle I_f M_f|V_\lambda|I_i M_i\rangle Y_{\lambda\mu}(\hat{R})$$

- In many interesting cases: $V_{\lambda\mu}(R,\xi) = \mathcal{F}_\lambda(R)\mathcal{T}^*_{\lambda\mu}(\xi)$

$$V(\mathbf{R},\xi) = \sqrt{4\pi}\sum_{\lambda}\mathcal{F}_\lambda(R)\sum_{\mu}\mathcal{T}^*_{\lambda\mu}(\xi)Y_{\lambda\mu}(\hat{R})$$

$$\langle I_f M_f|V(\mathbf{R},\xi)|I_i M_i\rangle = \sqrt{4\pi}\sum_{\lambda}\mathcal{F}_\lambda(R)\sum_{\mu}\langle I_f M_f|\mathcal{T}^*_{\lambda\mu}(\xi)|I_i M_i\rangle Y_{\lambda\mu}(\hat{R})$$

Remainder of Wigner-Eckart theorem: reduced matrix elements

$$\langle I_f M_f|\hat{O}_{\lambda\mu}|I_i M_i\rangle = C(I_i, I_f, \lambda)\langle I_f M_f|\lambda\mu I_i M_i\rangle \underbrace{\langle I_f||\hat{O}_\lambda||I_i\rangle}_{\text{r.m.e}}$$

Two popular conventions in Nuclear Physics:

1. Bohr-Mottelson (BM) convention: $C(I_i, I_f, \lambda) = (2I_f + 1)^{-1/2}$

$$\langle I_f M_f|\hat{O}_{\lambda\mu}|I_i M_i\rangle = (2I_f + 1)^{-1/2}\langle I_f M_f|\lambda\mu I_i M_i\rangle\langle I_f||\hat{O}_\lambda||I_i\rangle_{\text{BM}}$$

2. Brink-Satchler (BS) convention: $C(I_i, I_f, \lambda) = (-1)^{2\lambda}$

$$\langle I_f M_f|\hat{O}_{\lambda\mu}|I_i M_i\rangle = (-1)^{2\lambda}\langle I_f M_f|\lambda\mu I_i M_i\rangle\langle I_f||\hat{O}_\lambda||I_i\rangle_{\text{BS}}$$

So, the r.m.e. are related by:

$$\langle I_f||\hat{O}_\lambda||I_i\rangle_{\text{BM}} = \sqrt{2I_f + 1}\langle I_f||\hat{O}_\lambda||I_i\rangle_{\text{BS}}$$

Coupling potentials in terms of reduced matrix elements

Using BM convention:

1. General case:

$$\langle I_f M_f | V(\mathbf{R}, \xi) | I_i M_i \rangle = \sqrt{4\pi} \sum_{\lambda} \frac{\langle I_f M_f | \lambda \mu I_i M_i \rangle}{\sqrt{2I_f + 1}} \underbrace{\langle I_f || V_\lambda(R, \xi) || I_i \rangle} Y_{\lambda\mu}(\hat{R})$$

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2. If the structure part can be singled out:

$$\langle I_f M_f | V(\mathbf{R},\xi) | I_i M_i \rangle = \sqrt{4\pi} \sum_\lambda \frac{\langle I_f M_f | \lambda\mu I_i M_i \rangle}{\sqrt{2I_f+1}} \underbrace{\mathcal{F}_\lambda(R) \langle I_f || \mathcal{T}^*_\lambda(\xi) || I_i \rangle} Y_{\lambda\mu}(\hat{R})$$

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Using BM convention:

1. General case:

$$\langle I_f M_f | V(\mathbf{R},\xi) | I_i M_i \rangle = \sqrt{4\pi} \sum_\lambda \frac{\langle I_f M_f | \lambda\mu I_i M_i \rangle}{\sqrt{2I_f+1}} \underbrace{\langle I_f || V_\lambda(R,\xi) || I_i \rangle} Y_{\lambda\mu}(\hat{R})$$

2. If the structure part can be singled out:

$$\langle I_f M_f | V(\mathbf{R},\xi) | I_i M_i \rangle = \sqrt{4\pi} \sum_\lambda \frac{\langle I_f M_f | \lambda\mu I_i M_i \rangle}{\sqrt{2I_f+1}} \underbrace{\mathcal{F}_\lambda(R) \langle I_f || \mathcal{T}^*_\lambda(\xi) || I_i \rangle} Y_{\lambda\mu}(\hat{R})$$

☞ *The structure information is condensed in the transition potentials:*

$$V^\lambda_{fi}(R) \equiv \langle I_f || V_\lambda(R,\xi) || I_i \rangle = \mathcal{F}_\lambda(R) \langle I_f || \mathcal{T}^*_\lambda(\xi) || I_i \rangle$$

Coupling potentials in the channel basis

⇨ For projectile excitations ⇒ $\beta \equiv \{n(LI)\}$ (channel):

$$V^J_{\beta,\beta'}(R) \rightarrow \langle (L_f I_f) J_T | V | (L_i I_i) J_T \rangle$$

⇨ Coupling potentials in channel basis:

$$\langle (L_f I_f) J_T | V | (L_i I_i) J_T \rangle = \sqrt{4\pi} \sum_\lambda (-1)^{\lambda + L_i + I_f + J_T} \times \begin{Bmatrix} L_i & I_i & J_T \\ I_f & L_f & \lambda \end{Bmatrix} \langle L_f || Y_\lambda || L_i \rangle \mathcal{F}_\lambda(R) \langle I_f || \mathcal{T}_\lambda || I_i \rangle$$

⇨ All structure (model) contained in the transition potentials

$$\boxed{V^\lambda_{fi}(R) = \mathcal{F}_\lambda(R) \langle I_f || \mathcal{T}_\lambda || I_i \rangle}$$

Models for inelastic scattering

Strategy

1. Identify the projectile-target interaction $V(\mathbf{R},\xi)$ consistent with our structure model.
2. Perform a multipole expansion:

$$V(\mathbf{R},\xi) = \sqrt{4\pi}\sum_{\lambda,\mu} V_{\lambda\mu}(R,\xi)Y_{\lambda\mu}(\hat{R})$$

3. Evaluate the matrix elements in the basis of internal states:

$$\langle I_f M_f|V(\mathbf{R},\xi)|I_i M_i\rangle = \sqrt{4\pi}\sum_{\lambda,\mu}\frac{\langle I_f M_f|\lambda\mu I_i M_i\rangle}{\sqrt{2I_f+1}}\langle I_f||V_\lambda(R,\xi)||I_i\rangle Y_{\lambda\mu}(\hat{R})$$

4. Identify the multipole transition potentials

$$V^\lambda_{fi}(R) \equiv \langle I_f||V_\lambda(R,\xi)||I_i\rangle = \mathcal{F}_\lambda(R)\langle I_f||\mathcal{T}^*_\lambda(\xi)||I_i$$

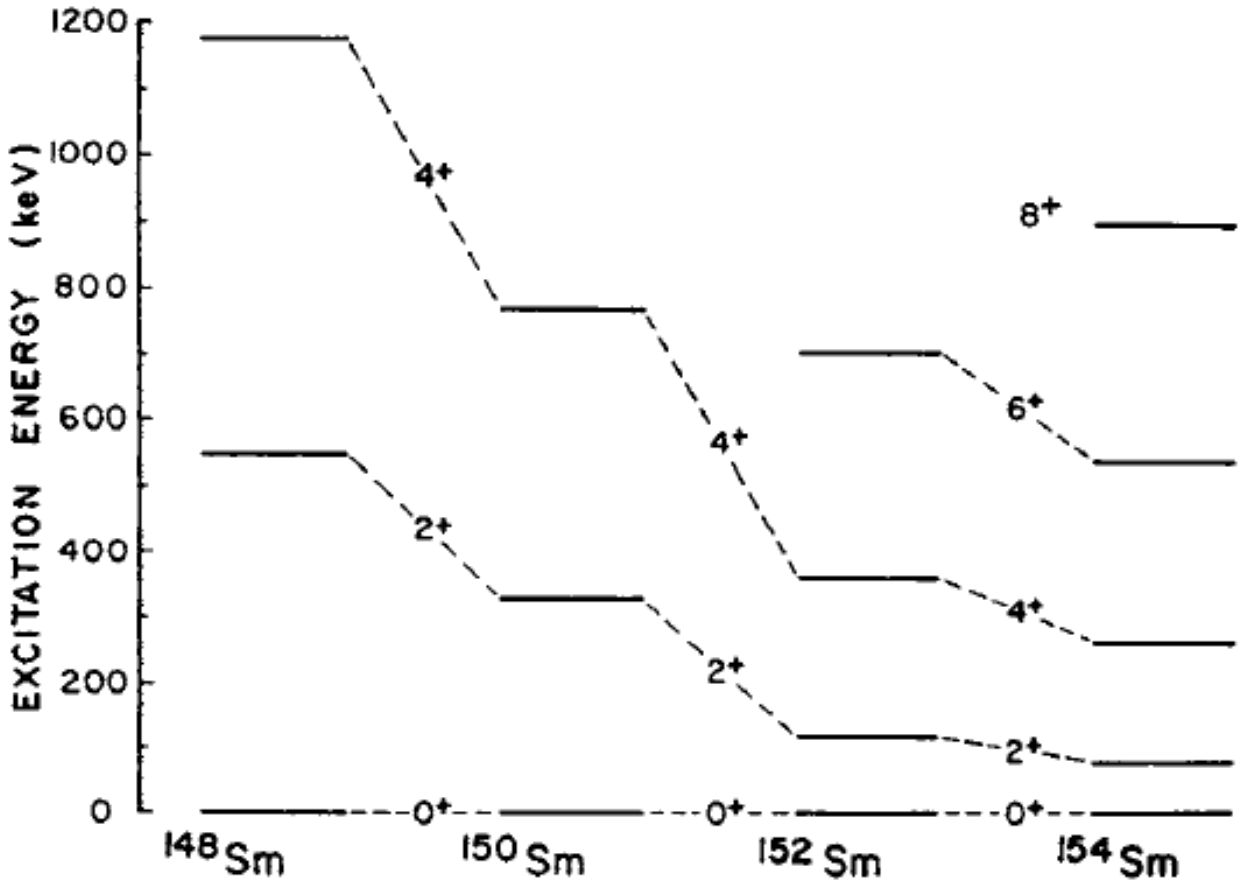
Types of collective excitations

The nucleons can move inside the nucleus in a coherent (collective) way.

1. Vibrations (spherical nuclei): small surface oscillations in shape.
2. Rotations (non-spherical nuclei): permanent deformation.
3. Monopole (*breathing*) mode: oscillations in the size (radius).
4. Isovector excitations (protons and neutrons move out of phase) (eg. giant dipole resonance)

Types of collective excitations

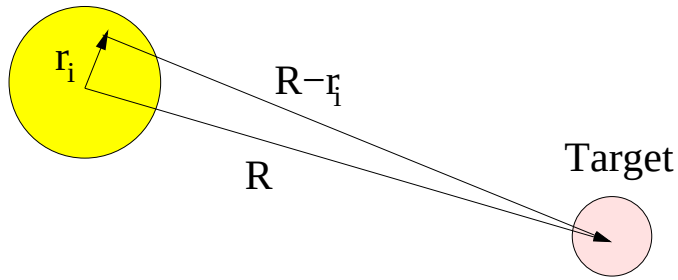
☞ The type of collective motion is closely related to the kind of energy spectrum.



Inelastic scattering: Coulomb excitation

- Projectile-target Coulomb interaction:

$$V(\mathbf{R},\xi) = \sum_i^{Z_p} \frac{Z_t e^2}{|\mathbf{R}-\mathbf{r}_i|}; \qquad \xi \equiv \{\mathbf{r}_i\}$$



- Multipolar expansion:

$$\frac{1}{|\mathbf{R}-\mathbf{r}_i|} = \sum_{\lambda\mu} \frac{4\pi}{2\lambda+1} \frac{r_i^\lambda}{R^{\lambda+1}} Y^*_{\lambda\mu}(\hat{r}_i) Y_{\lambda\mu}(\hat{R}) \qquad (R > r_i)$$

- Electric multipole operator: $M(E\lambda,\mu) \equiv e \sum_i r_i^\lambda Y^*_{\lambda\mu}(\hat{r}_i)$

Inelastic scattering: Coulomb excitation

- Monopole and transition operator:

$$V(\mathbf{R},\xi) = V_0(R) + \Delta(\mathbf{R},\xi) = \frac{Z_t Z_p e^2}{R} + 4\pi Z_t e \sum_{\lambda>0,\mu} \frac{M(E\lambda,\mu)}{2\lambda+1} \frac{Y_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}$$

- Transition potentials:

$$V_{nm}(\mathbf{R}) = \sum_{\lambda>0,\mu} \frac{4\pi}{2\lambda+1} \frac{Z_t e}{R^{\lambda+1}} \langle n I_n M_n | M(E\lambda,\mu) | m I_m M_m \rangle Y_{\lambda\mu}(\hat{R})$$

- Wigner-Eckart $\Rightarrow$ Multipolar transition potentials:

$$V^{\lambda}_{nm}(R) = \frac{\sqrt{4\pi}}{2\lambda+1} \frac{Z_t e}{R^{\lambda+1}} \langle n; I_n || M(E\lambda,\mu) || m; I_m \rangle$$

DWBA expressions for Coulomb excitation

DWBA scattering amplitude for a transition of multipolarity λ:

$$f(\mathbf{K}_f,\mathbf{K}_i)_{iM_i\to fM_f} = -\frac{\mu}{2\pi\hbar^2}\frac{4\pi Z_t e}{2\lambda+1}\langle f; I_f M_f|M(E\lambda,\mu)|i; I_i M_i\rangle$$
$$\times\int d\mathbf{R}\tilde{\chi}_f^{(-)}(\mathbf{K}_f,\mathbf{R})\frac{Y_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}\tilde{\chi}_i^{(+)}(\mathbf{K}_i,\mathbf{R})$$

Cross sections:

$$\left(\frac{d\sigma}{d\Omega}\right)_{iM_i\to fM_f} = \frac{K_f}{K_i}\left|f(\mathbf{K}_f,\mathbf{K}_i)_{iM_i\to fM_f}\right|^2$$

Unpolarized cross section (i.e. M_i and M_f not measured):,

$$\left(\frac{d\sigma}{d\Omega}\right)_{i\to f} = \frac{1}{(2I_i+1)}\frac{K_f}{K_i}\sum_{M_i,M_f}\left|f(\mathbf{K}_f,\mathbf{K}_i)_{iM_i\to fM_f}\right|^2$$

What can we learn measuring Coulomb excitation?

- For a inelastic excitation $i \to f$ of multipolarity λ the differential cross section is proportional to the electric transition probability $B(E\lambda; I_i \to I_f)$ because:.

$$B(E\lambda; i \to f) \equiv \frac{1}{2I_f + 1} |\langle f\ I_f || M(E\lambda) || i\ I_i \rangle|^2$$

$$\Downarrow$$

$$\frac{d\sigma}{d\Omega} \propto |\langle f\ I_f || M(E\lambda) || i\ I_i \rangle|^2 \propto B(E\lambda; I_i \to I_f)$$

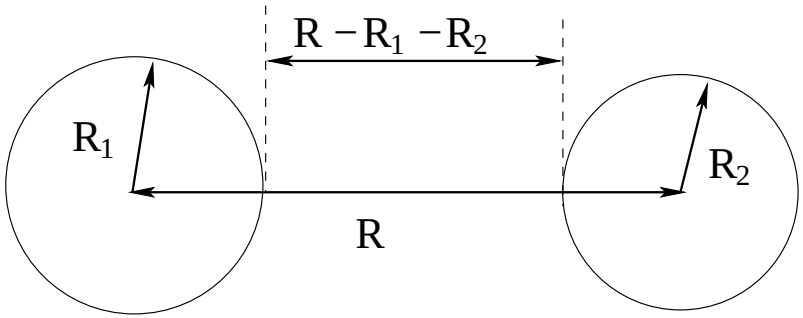
- *If the approximations involved in the derivation of the DWBA approximation are valid, the transition probabilities $B(E\lambda; I_f \to I_f)$ can be obtained comparing the magnitude of the inelastic cross sections with DWBA calculations.*

Nuclear collective excitations

- Central potential: Typically $U_{\rm nuc}(\mathbf{R}) = V(R - R_0), \quad R_0 = R_1 + R_2.$

☞ Eg: Woods-Saxon parametrization

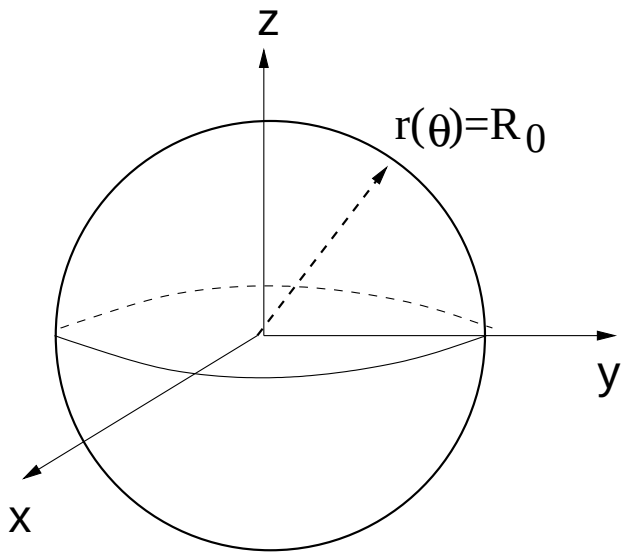
$$U_{\rm nuc}(R) = -\frac{V_0}{1+\exp\left(\frac{R-R_0}{a_r}\right)} - i\frac{W_0}{1+\exp\left(\frac{R-R_i}{a_i}\right)}$$



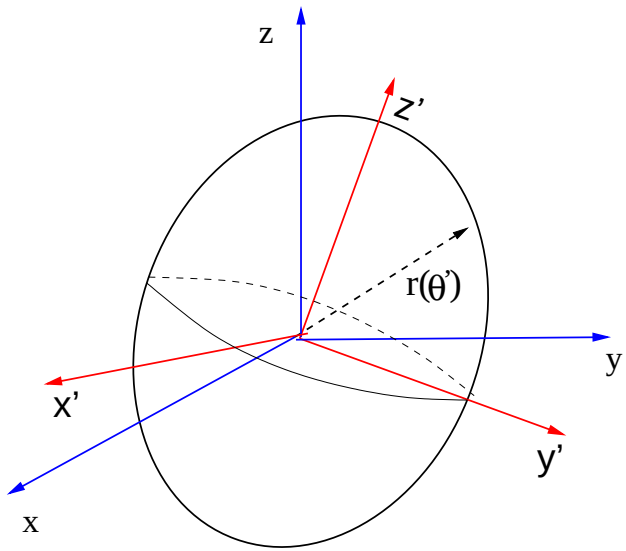
Collective excitations of a deformed nucleus (rotor model)

Axial deformed nucleus characterized by β_λ=deformation parameters

Spherical nucleus ($\beta = 0$)



Deformed nucleus ($\beta \neq 0$)



$$r(\theta') = R_0 \left[1 + \beta_2 Y_{20}(\theta', 0)\right]$$

Surface of a deformed nucleus

- Deformed nucleus with axial symmetry: $r(\theta') = R_0 \left[1 + \beta_2 Y_{20}(\theta', 0)\right]$

  $\delta_2 = \beta_2 R_0$ = (quadrupole) deformation length
- $Y_{20}(\theta', \phi')$ can be transformed to the laboratory frame:

$$Y_{\lambda 0}(\theta', 0) = \sum_{\mu} \mathcal{D}^{\lambda}_{\mu 0}(\alpha, \beta, \gamma) Y_{\lambda \mu}(\theta, \phi)$$

$\mathcal{D}^{\lambda}_{\mu,\mu'}$=rotation matrix α, β, γ=Euler angles.

- Define **deformation length operator:**

$$\hat{\delta}_{2\mu} \equiv \delta_2 \mathcal{D}^{2}_{\mu 0}(\alpha, \beta, \gamma) = \beta_2 R_0 \mathcal{D}^{2}_{\mu 0}(\alpha, \beta, \gamma)$$

- Deformed surface in LAB frame:

$$r(\theta, \phi) = R_0 + \sum_{\mu} \hat{\delta}_{2\mu} Y_{2,\mu}(\theta, \phi)$$

Deformed potential

- Deformed potential in intrinsic frame:

$$V(R - R_0) \rightarrow V(R - r(\theta', \phi')) \equiv V(\mathbf{R}, \xi)$$

- Multipole expansion:

$$V(R, \theta', \phi') = \sqrt{4\pi} \sum_{\lambda} V_\lambda(R) Y_{\lambda 0}(\theta', \phi')$$

$$V_\lambda(R) = \frac{1}{\sqrt{4\pi}} \int V(R - r(\theta', \phi')) Y^*_{\lambda\mu}(\theta', \phi')$$

- Deformed potential in LAB frame:

$$V(R, \theta, \phi) = \sqrt{4\pi} \sum_{\lambda\mu} V_\lambda(R) \mathcal{D}^\lambda_{\mu 0}(\alpha, \beta, \gamma) Y_{\lambda\mu}(\hat{R})$$

Transition potentials

- Internal states:

$$\langle \xi | K\ I\ M \rangle = \frac{\hat{I}}{\sqrt{8\pi^2}} \mathcal{D}^I_{MK}(\alpha, \beta, \gamma)$$

- Coupling potentials:

$$V_{if}(\mathbf{R}) \equiv \langle K\ I_f M_f | V | K\ I_i M_i \rangle = \sqrt{4\pi} \sum_{\lambda,\mu} V_\lambda(R) \langle K\ I_f M_f | \mathcal{D}^\lambda_{\mu 0} | K\ I_i\ M_i \rangle Y_{\lambda\mu}(\hat{R})$$

- Multipolar transition potentials:

$$V^\lambda_{fi}(R) = V_\lambda(R) \langle K\ I_f || \mathcal{D}^\lambda || K\ I_i \rangle$$

- Structure reduced matrix element: PROVE IT!

$$\langle K\ I_f || \mathcal{D}^\lambda || K\ I_i \rangle_{\rm BM} = \hat{I}_i \langle I_i\ K\ \lambda\ 0 | I_f\ K \rangle$$

Small deformations

- Multipole expansion of the potential (intrinsic frame):

$$V(\mathbf{R},\theta',\phi') = V_0(R-R_0) - \sum_\lambda \delta_\lambda \frac{dV_0(R-R_0)}{dR} Y_\lambda(\theta',\phi') + \ldots$$

$$V_\lambda(R) = -\frac{1}{\sqrt{4\pi}} \delta_\lambda \frac{dV_0(R-R_0)}{dR}$$

- Transition potentials for a multipole λ:

$$V^\lambda_{if}(R) - \frac{1}{\sqrt{4\pi}} \frac{dV_0}{dR} \delta_\lambda \langle K\ I_f || \mathcal{D}^\lambda || K\ I_i\rangle = -\frac{1}{\sqrt{4\pi}} \frac{dV_0}{dR} \langle K\ I_f || \hat{\delta}_\lambda || K\ I_i\rangle$$

☞*For small deformations, the nuclear transition potentials are proportional to the matrix element of the deformation length operator*.

DWBA amplitude

DWBA scattering amplitude:

$$f(\mathbf{K}',\mathbf{K})_{f,i} = -\frac{\mu}{2\pi\hbar^2}\langle f I_f M_f|\hat{\delta}_\lambda|i I_i M_i\rangle \int d\mathbf{R}\tilde{\chi}_f^{(-)}(\mathbf{K}',\mathbf{R})\frac{dV}{dR}Y_{\lambda\mu}(\theta,\phi)\tilde{\chi}_i^{(+)}(\mathbf{K},\mathbf{R})$$

Cross sections:

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{i\to f} = \frac{K_f}{K_i}\left|\langle f\ I_f M_f|\hat{\delta}_\lambda|i\ I_i M_i\rangle\right|^2\left(\frac{\mu}{2\pi\hbar^2}\right)^2 \times\left|\int d\mathbf{R}\tilde{\chi}_f^{(-)}(\mathbf{K}',\mathbf{R})\frac{dV}{dR}Y_{\lambda\mu}(\theta,\phi)\tilde{\chi}_i^{(+)}(\mathbf{K},\mathbf{R})\right|^2$$

- *The differential cross section is proportional to the deformation parameters*
- *If the approximations are valid, the deformation parameters can be obtained comparing the magnitude of the inelastic cross sections with DWBA calculations.*

Rotor model for the Coulomb excitation

For Coulomb excitation we had:

$$V_{if}(\mathbf{R}) = \sum_{\lambda>0} \frac{4\pi Z_t e}{2\lambda+1} \langle f; I_f M_f | M(E\lambda,\mu) | i; I_i M_i \rangle \frac{Y_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}$$

If the nucleus has a permanent deformation in intrinsic frame:

$$\langle K\ I_f || M(E\lambda) || K\ I_i \rangle = M_n(E\lambda) \hat{I}_i \langle I_i K \lambda 0 | I_f K \rangle$$

⇨ $M_n(E\lambda)$ is the reduced matrix element of the (charge) deformation in the intrinsic frame.

⇨ $M_n(E\lambda)$ is the same for all transition within a given rotational band (K)

Summary of physical ingredients for collective excitations

- Coulomb excitation → electric reduced matrix elements

$$V_{if}(\mathbf{R}) = \sum_{\lambda>0} \frac{4\pi Z_t e}{2\lambda+1} \langle f; I_f M_f | M(E\lambda,\mu) | i; I_i M_i \rangle \frac{Y_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}$$

- Nuclear excitation (collective model) → deformation lengths

$$V_{if}(\mathbf{R}) = -\frac{dV_0}{dR} \sum_{\lambda} \langle f\ I_f M_f | \hat{\delta}_{\lambda\mu} | i\ I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

Summary of physical ingredients for collective excitations

- Coulomb excitation → electric reduced matrix elements

$$V_{if}(\mathbf{R}) = \sum_{\lambda>0} \frac{4\pi Z_t e}{2\lambda+1} \langle f; I_f M_f | M(E\lambda,\mu) | i; I_i M_i \rangle \frac{Y_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}$$

- Nuclear excitation (collective model) → deformation lengths

$$V_{if}(\mathbf{R}) = -\frac{dV_0}{dR} \sum_{\lambda} \langle f\ I_f M_f | \hat{\delta}_{\lambda\mu} | i\ I_i M_i \rangle Y_{\lambda\mu}(\hat{R})$$

☞ *Within the rotational model: :*

⇨ $\boxed{\langle K\ I_f || M(E\lambda) || K\ I_i \rangle = M_n(E\lambda) \hat{I}_i \langle I_i K \lambda 0 | I_f K \rangle}$

⇨ $\boxed{\langle K\ I_f || \widehat{\delta_\lambda} || i\ K\ I_i \rangle = \delta_\lambda \hat{I}_i \langle I_i K \lambda 0 | I_f K \rangle}$ $\qquad \delta_\lambda = \beta_\lambda R$

Coulomb + nuclear potential

- We expect the Coulomb excitation to be more important when:
  - The projectile and/or target charges are large (i.e. large $Z_1 Z_2 \gg 1$)
  - At energies below the Coulomb barrier (where nuclear effects are less important).
  - At very forward angles (large impact parameters).

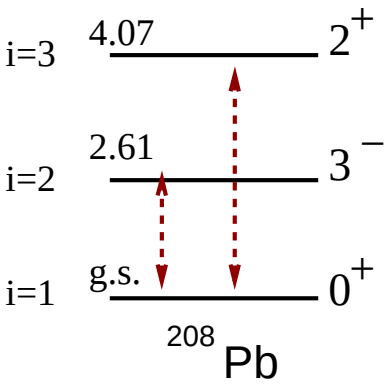
- If both Coulomb and nuclear contributions are important the scattering *amplitudes* for both processes should be added:

$$\left(\frac{d\sigma}{d\Omega}\right)_{i\to f} = \frac{K_f}{K_i}\left|f_{if}^{\rm coul} + f_{if}^{\rm nucl}\right|^2$$

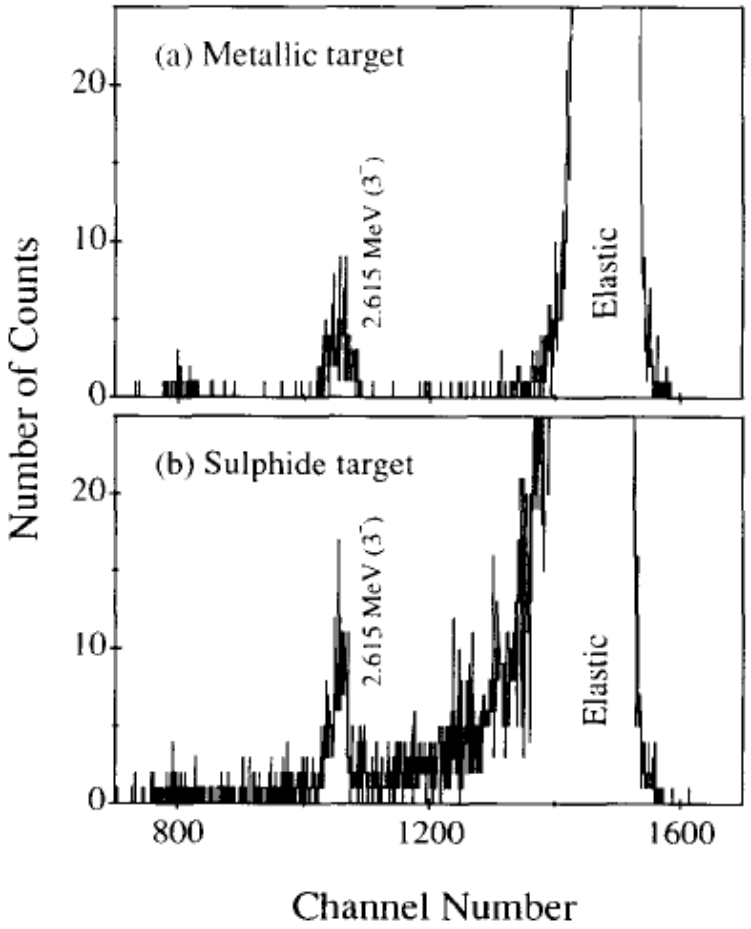
☞ *In this case, interferences effects will appear!*

Collective excitations: example

Physical example: $^{16}O + {}^{208}Pb \rightarrow {}^{16}O + {}^{208}Pb(3^-, 2^+)$



Outgoing ^{16}O energy:



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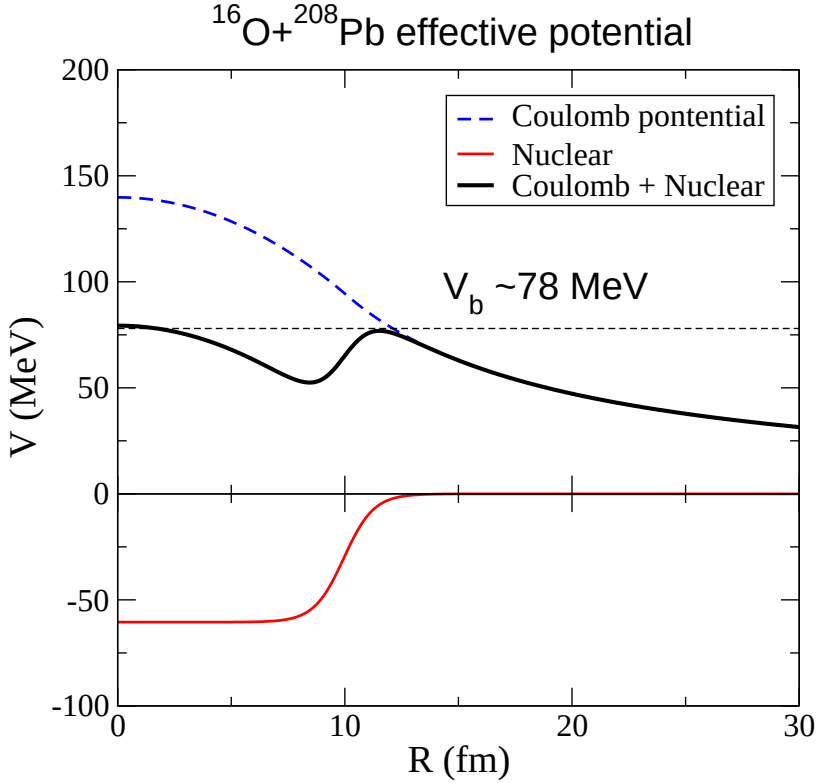
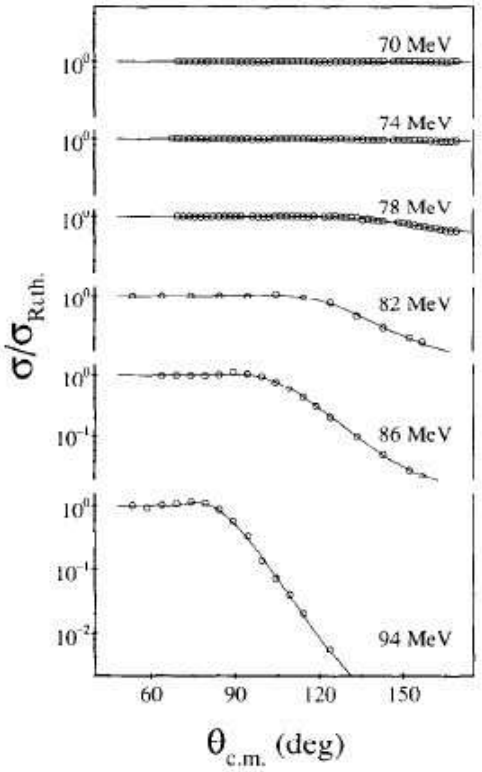
^{16}O+^{208}Pb effective interaction

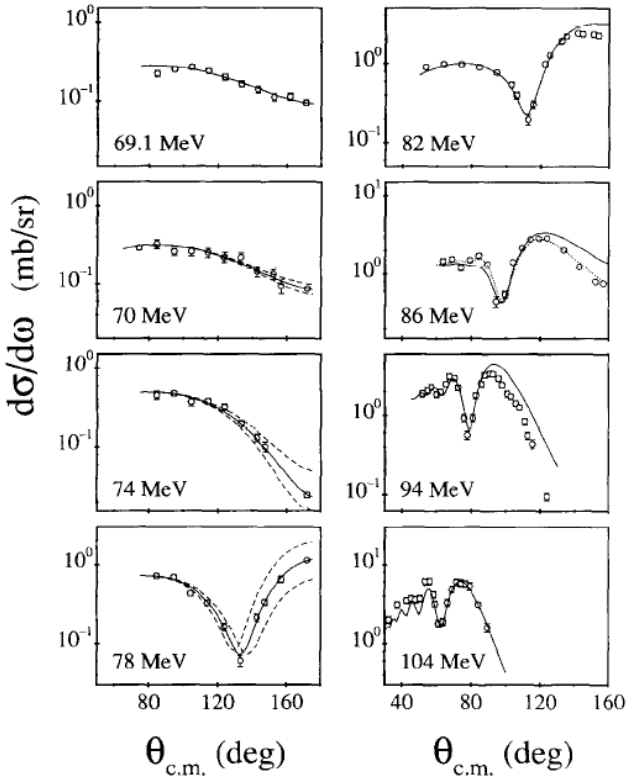


☞ Coulomb barrier:

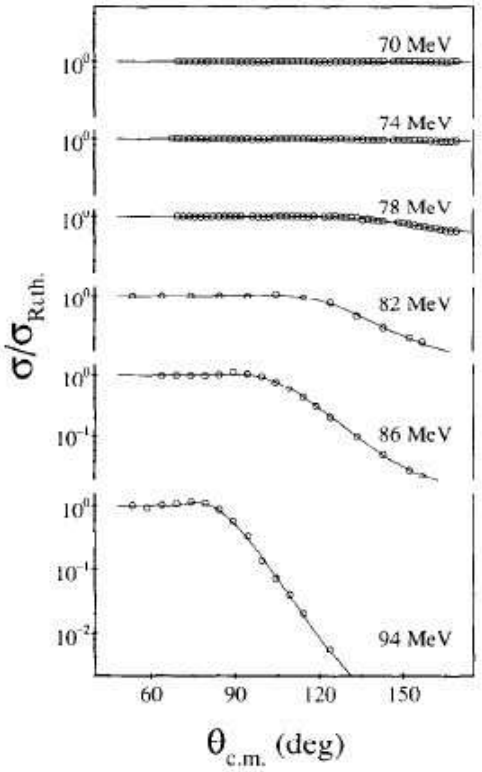
$$V_{\text{barrier}} \approx \frac{Z_p Z_t e^2}{1.44(A_p^{1/3} + A_t^{1/3})} \simeq 78 \text{ MeV}$$

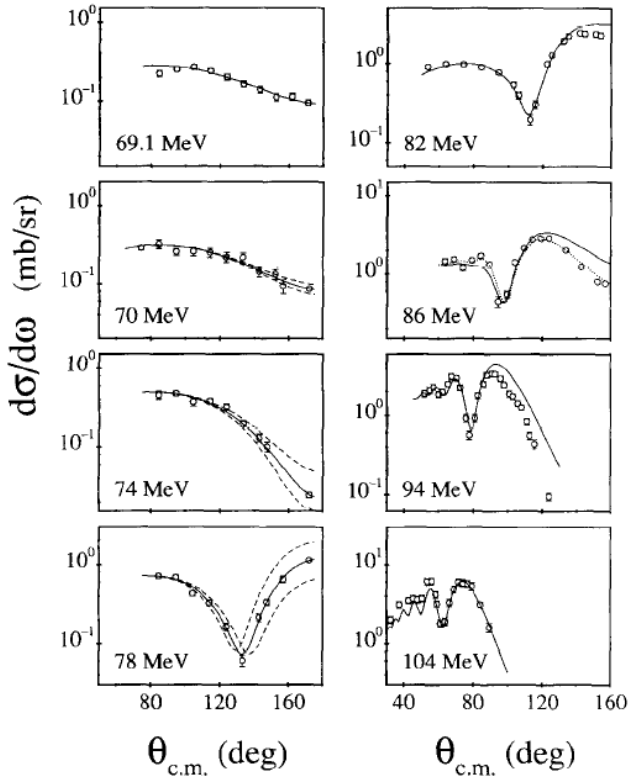
Collective excitations: example





Collective excitations: example



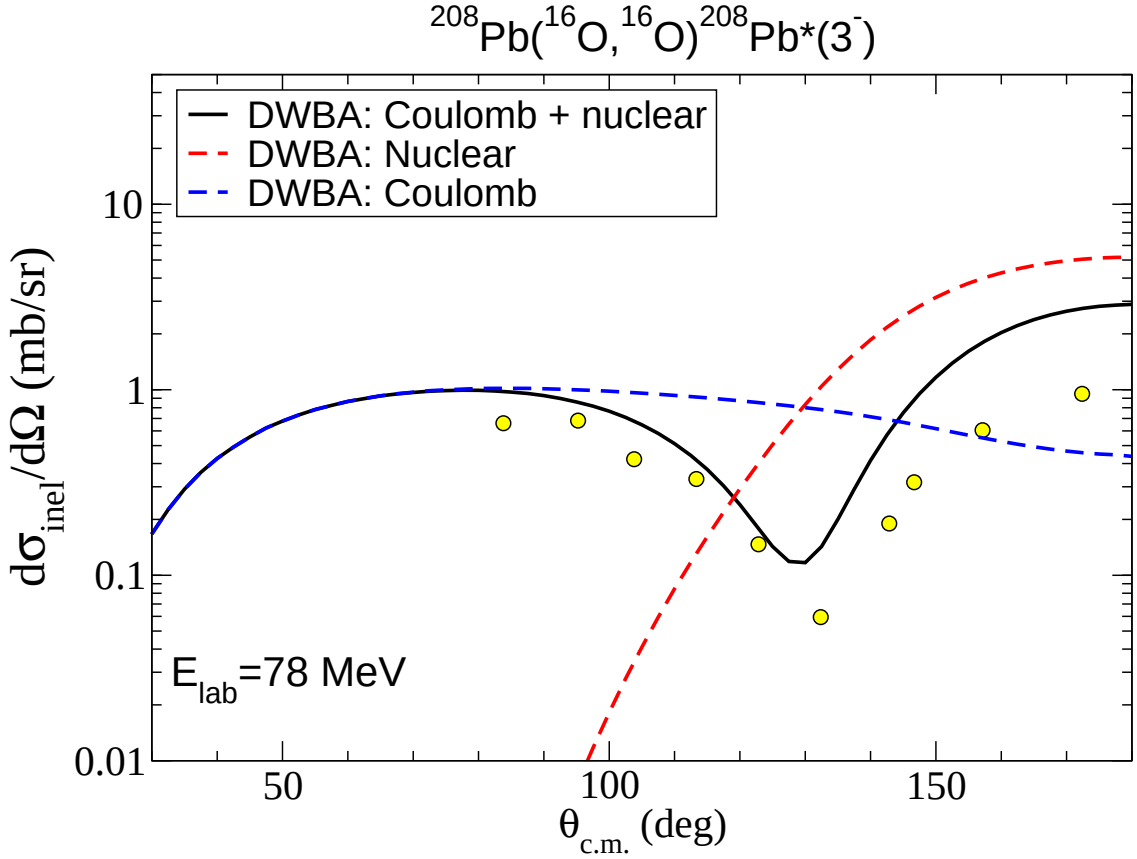


☞ Coulomb barrier:

$$V_{\text{barrier}} \approx \frac{Z_p Z_t e^2}{1.44(A_p^{1/3} + A_t^{1/3})} \simeq 78 \text{ MeV}$$

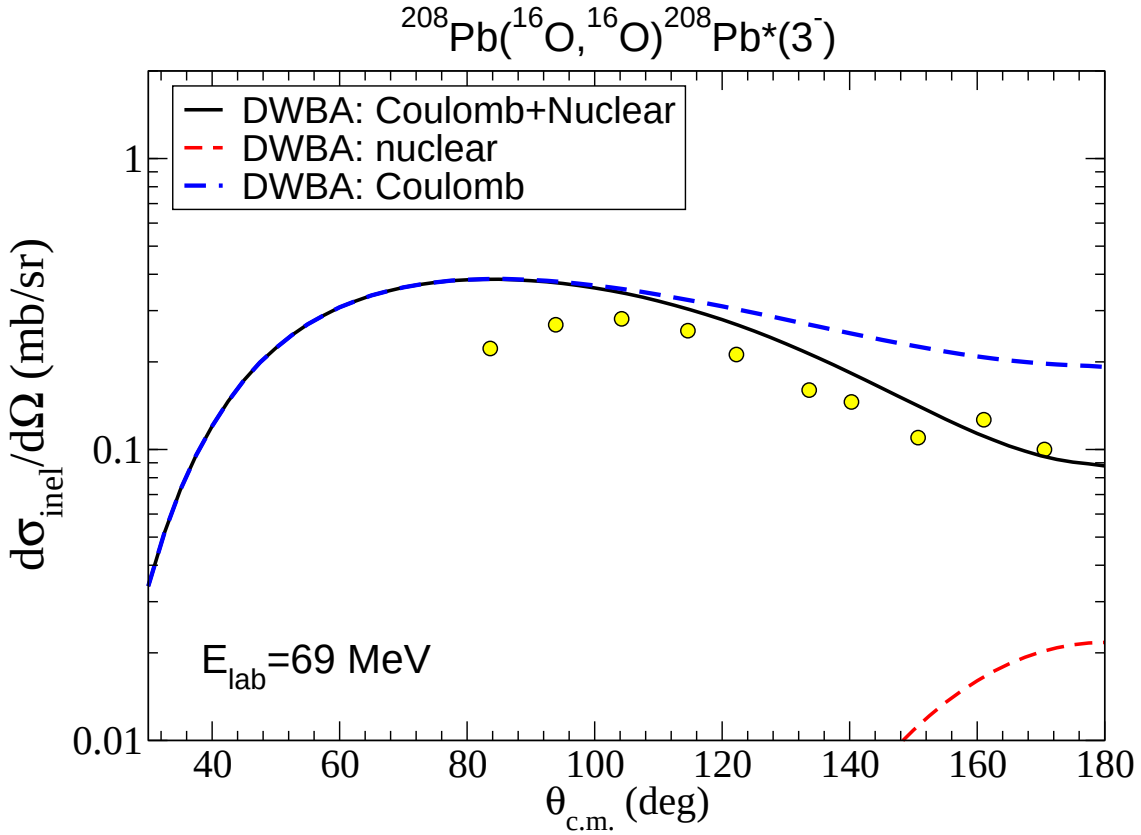
^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Coulomb and Nuclear excitations can produce constructive or destructive interference:



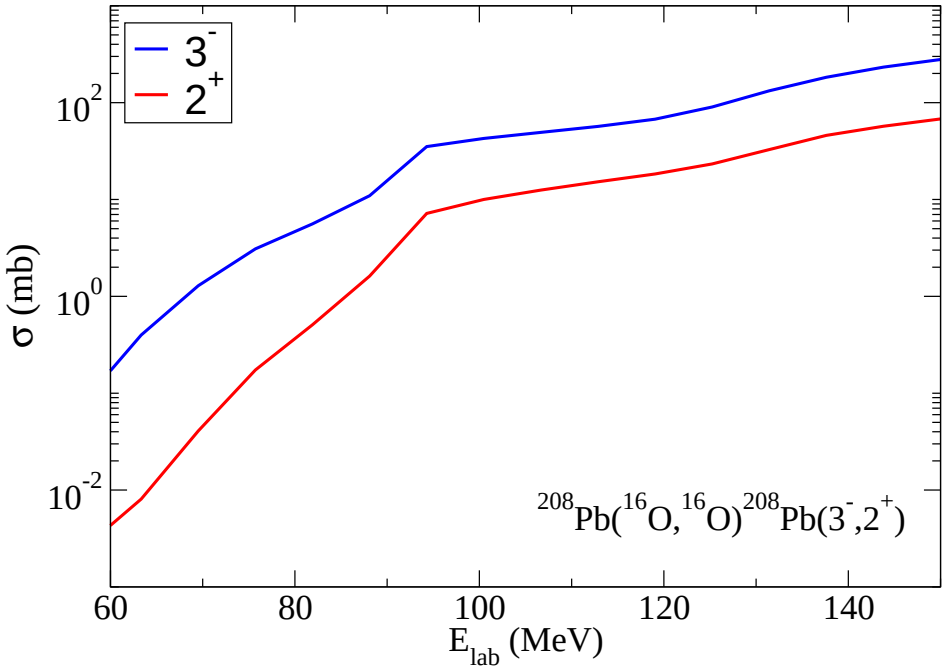
^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Below the barrier, the Coulomb excitation is dominant, and the interference is smaller:



^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Effect of the incident energy:



When running your calculations...

- Do not confuse δ_λ (deformation parameter in intrinsic frase) with $\hat{\delta}_{\lambda,\mu}$ (deformation length operator in LAB frame):
  - δ_λ is a c-number and is the same for all transitions within the same rotational band
  - $\hat{\delta}_{\lambda,\mu}$ is characterized by its matrix elements, which are different for each transition.
- When using $\langle I_f||\hat{\delta}_\lambda||I_i\rangle$ from literature, make sure what convention is used for the Wigner-Eckart theorem.
- FRESCO uses Bohr-Mottelson convention so

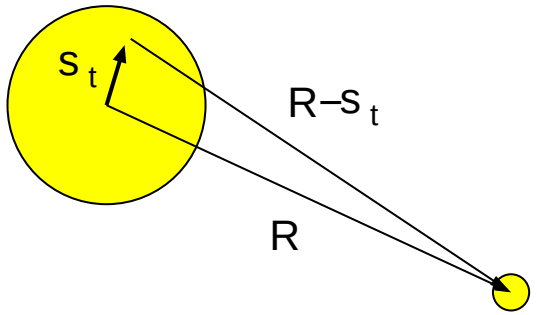
$$\langle I_f||\hat{\delta}_\lambda||I_i\rangle = \langle I_i||\hat{\delta}_\lambda||I_f\rangle$$

Microscopic folding models

1. Single-folding potential:

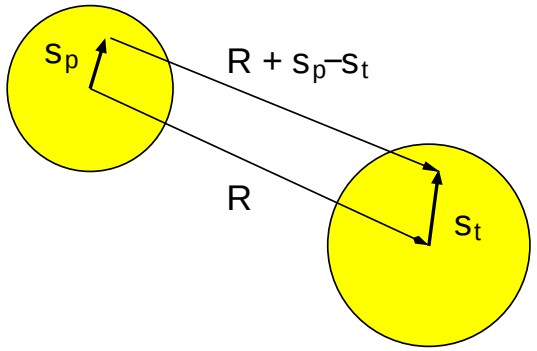
$$V(\mathbf{R}) = \int \rho_t(\mathbf{s}_t) v_{NN}(|\mathbf{R} - \mathbf{s}|) d\mathbf{s}$$

☞ $\rho_t(\mathbf{s}_t)$=target g.s. density.



2. Double-folding potential:

$$V(\mathbf{R}) = \int \rho_p(\mathbf{s}_p)\rho_t(\mathbf{s}_t) v_{NN}(|\mathbf{R} + \mathbf{s}_p - \mathbf{s}_t|) d\mathbf{s}_p d\mathbf{s}_t$$



Diagonal and transition densities

- Density operator:

$$\hat{\rho}(\mathbf{s},\xi) = \sum_{i=1}^{A} \delta(\mathbf{s}-\mathbf{r}_i) \qquad \xi \equiv \{\mathbf{r}_i\}$$

- Ground-state density:

$$\hat{\rho}_{\rm gs}(\mathbf{s}) \equiv \langle gs|\hat{\rho}(\mathbf{s},\xi)|gs\rangle = \langle \phi_0^*(\mathbf{r}_1,\ldots,\mathbf{r}_A)|\hat{\rho}(\mathbf{s},\xi)|\phi_0(\mathbf{r}_1,\ldots,\mathbf{r}_A)\rangle$$

- Transition density:

$$\langle j|\hat{\rho}(\mathbf{s},\xi)|i\rangle = \langle \phi_j^*(\mathbf{r}_1,\ldots,\mathbf{r}_A)|\hat{\rho}(\mathbf{s},\xi)|\phi_i(\mathbf{r}_1,\ldots,\mathbf{r}_A)\rangle$$

- Multipole expansion of density operator:

$$\hat{\rho}(\mathbf{s},\xi) = \sqrt{4\pi}\sum_{\lambda\mu} \hat{\rho}_{\lambda\mu}(s,\xi) Y_{\lambda\mu}(\hat{s})$$

Relation to physical quantities:

- $\lambda = 0 \Rightarrow$ matter densities:

$$\int \rho_{00}(\mathbf{s})d\mathbf{s} = A; \quad \int \rho_{00}^{(p)}(\mathbf{s})d\mathbf{s} = Z; \quad \int \rho_{00}^{(n)}(\mathbf{s})d\mathbf{s} = N$$

- $\lambda > 0 \Rightarrow$ electric moments, transition probabilities,etc

$$\langle I_f||E\lambda||I_i\rangle = \int_0^\infty r^\lambda \langle I_f||\rho_\lambda^{(p)}||I_i\rangle r^2 dr$$

$$B(E\lambda; I_i \rightarrow I_f) = (2I_i+1)^{-1/2} e^2 \left|\langle I_f||E\lambda||I_i\rangle\right|^2; \quad i \neq f$$

$$Q_2 = \sqrt{16\pi/5}\,(2I+1)^{-1/2}\,\langle II20|II\rangle\,\langle I||E2||I\rangle \quad I_i = I_f = I > 0$$

Multipole expansion of the potential

$$V(\mathbf{R},\xi) = \sqrt{4\pi}\sum_{\lambda\mu}\int \hat{\rho}_{\lambda\mu}(s,\xi)Y_{\lambda\mu}(\hat{s})v_{NN}(|\mathbf{R}-\mathbf{s}|)d\mathbf{s}$$

Comparing with the general expansion,

$$V(\mathbf{R},\xi) = \sqrt{4\pi}\sum_{\lambda\mu} V_{\lambda\mu}(R,\xi)Y_{\lambda\mu}(\hat{R})$$

one gets

$$\boxed{V_{\lambda\mu}(R,\xi) = \int \hat{\rho}_{\lambda\mu}(s,\xi)v_{NN}(|\mathbf{R}-\mathbf{s}|)P_\lambda(\cos(\theta))d\mathbf{s}} \qquad \cos(\theta) = \hat{r}\cdot\hat{s}$$

$$\boxed{\langle I_f||V_\lambda(R)||I_i\rangle = V_{fi}^{\lambda} \equiv \int \langle I_f||\hat{\rho}_\lambda||I_i\rangle v_{NN}(|\mathbf{R}-\mathbf{s}|)P_\lambda(\cos(\theta))d\mathbf{s}}$$

Input examples for inelastic scattering

Remainder of some important formulae

The main physical ingredient of CC and DWBA calculations are the coupling potentials:

$$V_{fi}^{\lambda}(R) \equiv \langle I_f \| V_\lambda(R,\xi) \| I_i \rangle = \mathcal{F}_\lambda(R) \langle I_f \| \mathcal{T}_\lambda^*(\xi) \| I_i \rangle$$

- $\mathcal{F}_\lambda(R)$=radial formfactor
- $\langle I_f \| \mathcal{T}_\lambda^*(\xi) \| I_i \rangle$=structure reduced matrix element

Reduced matrix elements for inelastic couplings

Coupling	TYPE	P(k)	STR
Coulomb (rotational)	10,11	$M_n(k)$	
Coulomb (general)	12,13	$\neq 0$ (arbitrary)	$M(Ek) = (-1)^{\frac{I-I'+\|I-I'\|}{2}} \langle I' \| M(Ek) \| I \rangle)$
Nuclear (rotational)	10,11	$DEF(k) = R\beta_k$	-
Nuclear (general)	12,13	$DEF(k) = R\beta_k$	$RDEF(k) = (-1)^{\frac{I-I'+\|I-I'\|}{2}} \langle I' \| \delta_k \| I \rangle$

Input example for inelastic scattering with collective form factors

Inelastic scattering within the rotor model: ^{10}Be + ^{1}H

Input example for ^{10}Be + ^{1}H

```
0Be+p @ 63.7 MeV/u DWBA (rotor model)
AMELIST
FRESCO hcm=0.05 rmatch=40.0
    jtmin=0.0 jtmax=50 absend=-1
    thmin=0.1 thmax=90.00
    iter=1 nnu=24
    smats=2
      xstabl=1
      elab=637.7 /

PARTITION namep='10Be' massp=10.0113 zp=4
        namet='p' masst=1.0078 zt=1 nex=2 /
STATES jp=0.0 bandp=1 ep=0.000 cpot=1 ep=0.0 jt=0.0 /
STATES jp=2.0 bandp=1 ep=3.368 cpot=1 copyt=1 /
partition /

POT kp=1 ap=10 at=0.0 rc=1.0 /
POT kp=1 type=10 shape=10 p2=7.8 /
POT kp=1 type=1 p1=31.64 p2=1.145 p3=0.69
                  p4=8.78 p5=1.134 p6=0.69 /
POT kp=1 type=10 shape=10 p2=1.97 /
step /

pot /

overlap /

coupling /
```

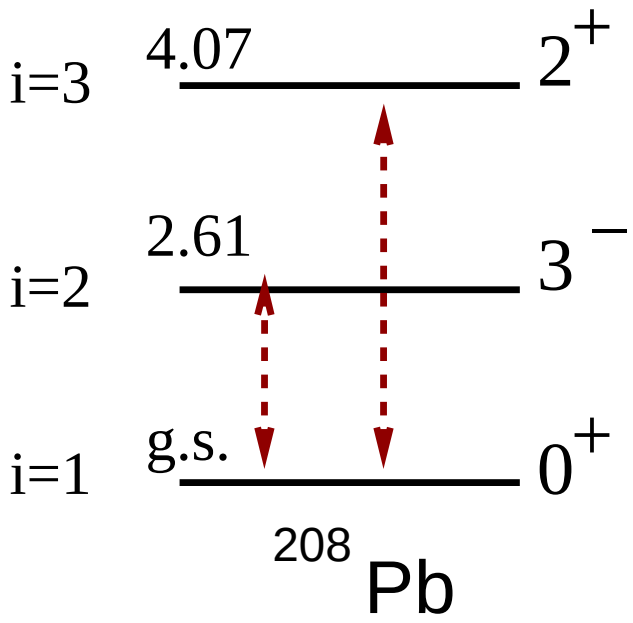
- `iter`=1: Solve CC to 1st order (DWBA)
- For the excited pair: `copyt=1`
- Coulomb deformation:

  `type`=10 (rotor)
  `shape`=10 (usual deformed charge distribution)
  `Mn`=7.8 e fm^2
- Nuclear deformation:

  `type`=10 (rotor)
  `shape`=10 (derivative of undeformed potential)
  δ_2=1.97 fm.

Collective excitations from general matrix elements

Example: ^{208}Pb(^{16}O,^{16}O)^{208}Pb(3^-,2^+)



Input example for ^{208}Pb(^{16}O,^{16}O)^{208}Pb(3^-,2^+)

Input example 2: ^{208}Pb(^{16}O,^{16}O)^{208}Pb(3^-,2^+) (o16pb-cc1a.in)

```
o16pb_cc1a.in: 16O+208Pb 80 MeV
NAMELIST
&FRESCO hcm=0.05 rmatch=100.0
        jtmin=0.0 jtmax=300.0
        thmin=5.00 thmax=-180.00 thinc=2.50
        iblock=3
        smats=2 xstabl=1 elab= 80.0 /

&PARTITION namep='16O' massp=15.99 zp=8
          namet='PB208' masst=207.98 zt=82
          nex=3 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1
       jt=0.0 bandt=1 et=0.0   /
&STATES jp=0.0 copyp=1 ep=0.0 cpot=1
       jt=3.0 bandt=-1 et=2.61 fexch=F /
&STATES jp=0.0 copyp=1 bandp=1 ep=0.0 cpot=1
       jt=2.0 bandt=1 et=4.07 /
&partition /
```

```
&POT kp=1 itt=F ap=208.000 at=16.000 rc=1.200 /
&POT kp=1 type=13 shape=10 itt=F p2=54.45 p3=815.0
/
&STEP ib=1 ia=2 k=3 str=815.0 /
&STEP ib=2 ia=1 k=3 str=815.0 /
&STEP ib=1 ia=3 k=2 str=54.45 /
&STEP ib=3 ia=1 k=2 str=54.45 /
&step /
&POT kp=1 type=1 shape=1
p4=10.000 p5=1.000 p6=0.400 /
&POT kp=1 type=-1 p1=60.500 p2=1.179 p3=0.658 /
&POT kp=1 type=13 shape=11 p2=0.400 p3=0.8 /
&STEP ib=1 ia=2 k=3 str=0.8 /
&STEP ib=2 ia=1 k=3 str=0.8 /
&STEP ib=1 ia=3 k=2 str=0.4 /
&STEP ib=3 ia=1 k=2 str=0.4 /
&step /

&pot /

&overlap /
&coupling /
```

^{208}Pb(^{16}O,^{16}O)^{208}Pb input example

General variables:

```
&FRESCO hcm=0.05 rmatch=100.0
       jtmin=0.0 jtmax=300.0
       thmin=5.00 thmax=-180.00 thinc=2.50
       iblock=3
       smats=2 xstabl=1
     elab= 80.0 /
```

`iblock`: Number of states (including gs) that will be coupled to all orders.

- `iblock=1`: only elastic scattering
- `iblock=2`: elastic scattering + 1st inelastic channel (^{208}Pb(3^-))
- `iblock=3`: elastic scattering + ^{208}Pb(3^-) + ^{208}Pb(2^+)

^{208}Pb(^{16}O,^{16}O)^{208}Pb input example

Partitions and states:

```
&PARTITION namep='16O' massp=15.99 zp=8 namet='208Pb' masst=207.98 zt=82
           nex=3 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=+1 et=0.00 /
&STATES      copyp=1          cpot=1 jt=3.0 bandt=-1 et=2.61 /
&STATES      copyp=1          cpot=1 jt=2.0 bandt=+1 et=4.07 /
&partition /
```

- `nex`: number of states within the partition
- `ep, et`: excitation energy for projectile / target
- `copyp=1` tells FRESCO that the 2nd and 3rd projectile states are just a copy of the ground state.

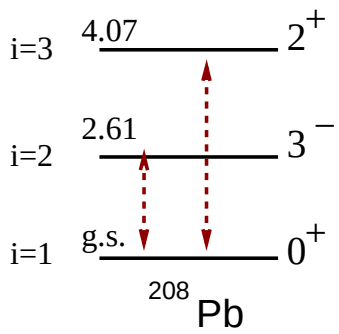
^{208}Pb(^{16}O,^{16}O)^{208}Pb input example

Coulomb excitation:

```
&POT kp=1 ap=208.000 at=16.000 rc=1.2 /
&POT kp=1 type=13 shape=10 p2=54.45 p3=815.0 p4=0 p5=0 p6=0 /
```

- `type=13`: couple target states by deforming previous potential
- `p1,...,p6`: consider couplings for multipoles k with `pk`$\neq 0$
- `shape=10`: usual deformed charge sphere: $V^k_{nm}(R) \propto M(Ek)/R^{k+1}$

```
&STEP ib=1 ia=2 k=3 str=815.0 /
&STEP ib=2 ia=1 k=3 str=815.0 /
&STEP ib=1 ia=3 k=2 str=54.45 /
&STEP ib=3 ia=1 k=2 str=54.45 /
&step /
```



- `ia, ib`: couple from state number `ia` to state `ib`
- `k`: multipolarity
- `str`=$\langle$ib$\|$M(Ek)$\|$ia$\rangle$ = $\sqrt{(2I_a+1)B(E\lambda; ia \rightarrow ib)}$

^{208}Pb(^{16}O,^{16}O)^{208}Pb input example

Nuclear excitation:

```
&POT kp=1 type=1 shape=1 p4=10.000 p5=1.000 p6=0.400 /
&POT kp=1 type=-1 shape=0 p1=60.500 p2=1.179 p3=0.658 /
&POT kp=1 type=13 shape=10 itt=F p2=0.400 p3=0.8 /
```

- `type=13`: couple target states by deforming preceding potential
- `shape=10`: usual deformed nuclear potential: $V^k_{nm}(R) \propto \delta_k dU(R)/dR$

```
&STEP ib=1 ia=2 k=3 str=0.8 /
&STEP ib=2 ia=1 k=3 str=0.8 /
&STEP ib=1 ia=3 k=2 str=0.4 /
&STEP ib=3 ia=1 k=2 str=0.4 /
```

- `str`=$\langle ib||\delta_k||ia\rangle$ (reduced deformation length)

^{208}Pb(^{16}O,^{16}O)^{208}Pb input example

Useful output files:

- Main output file:

```
0CUMULATIVE REACTION cross section                    =    11.22270  <L> =     47.07
0CUMULATIVE outgoing cross sections in partition 1 :     0.00000    7.67943     0
0Cumulative ABSORBTION by Imaginary Potentials        =     2.55189  <L> =      6.99
```

- Angular distributions:
  - `fort.201` : Elastic scattering angular distribution
  - `fort.202` : 1st state angular distribution
  - `fort.203` : 2nd excited state angular distribution

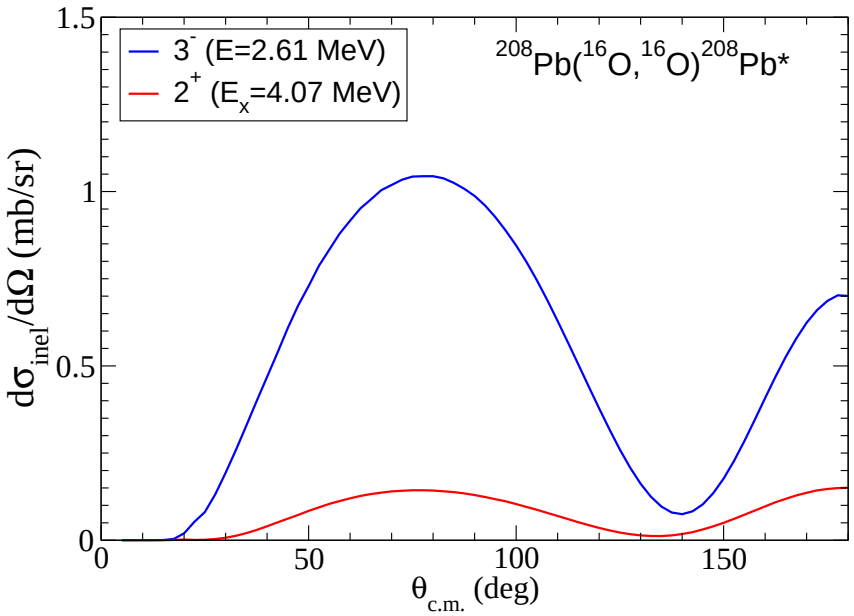
- `fort.56`: 3 columns: Fusion (absorption), reaction and inelastic cross section for each total angular momentum J.

$$\sigma_{\rm reac} = \sigma_{\rm inel} + \sigma_{\rm abs}$$

^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Angular distribution of the ejectile in c.m. frame

Absolute cross section



Ratio to elastic cross section