Training in Advanced Low Energy Nuclear Theory
Semiclassical methods for Coulomb excitation

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Introduction

1. Semiclassical methods
   - Classical trajectories
   - Time-dependent solution of the scattering problem
   - Application to Coulomb excitation
   - Application to Coulomb dissociation of halo nuclei
   - Higher-order effects
   - Relation to radiative capture

The scale of the nucleus

- Typical size of the nucleus: $R \sim 5 \times 10^{-15}$ m
- Typical energy of a nucleon in a nucleus: $E \sim 30$ MeV ($v \sim c/5$)
- De Broglie associated wavelength:

$$\lambdabar = \frac{\hbar}{p} = \frac{\hbar}{m \cdot v} \sim \frac{5\hbar c}{m c^2} \approx 1\,\text{fm}$$

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For a particle moving with momentum p: $\lambda\!\!\!^{-} = \hbar/p$

TABLE 2.1 *Reduced de Broglie wavelengths* $\lambda\!\!\!^{-}$*, in fm, for various particles and energies*

Energy	*Photon*	*Electron*	*Pion*	*Proton*	*α-Particles*	^{16}O	^{40}Ar	^{208}Pb
1 MeV	197	140	12	4.5	2.3	1.14	0.72	0.32
10 MeV	19.7	18.7	3.7	1.4	0.72	0.36	0.23	0.10
100 MeV	2.0	2.0	1.0	0.45	0.23	0.11	0.072	0.032
1 GeV	0.20	0.20	0.17	0.12	0.068	0.035	0.023	0.010

Classical vs semi-classical scattering?

- CLASSICAL SCATTERING: classical particles moving along classical trajectories

$$\lambdabar = \frac{\hbar}{m \cdot v} \ll R$$

- SEMI-CLASSICAL SCATTERING: quantum particles moving along classical trajectories.

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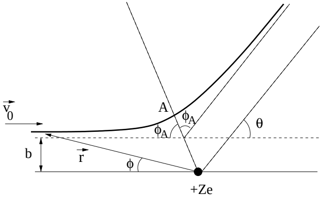
☞ To define a classical trajectory with a given velocity v, one must assume that the relative motion is not much affected by the internal excitations of the projectile, i.e.:

$$\boxed{\frac{\Delta E_n}{E} \ll 1} \quad \text{and} \quad \boxed{\frac{\Delta L_n}{L} \ll 1}$$

Description of a classical trajectory

A classical trajectory can be characterized by the polar variables (r, ϕ).



The equation of the trajectory is obtained from:

- Energy conservation:

$$E = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{2\mu r^2} + V(r) = \frac{1}{2}\mu v_0^2$$

- Angular momentum conservation:

$$L = \mu r^2 \frac{d\phi}{dt} = \mu \cdot v_0 \cdot b = \text{cte}$$

Equation of the trajectory

$$\frac{d\phi}{dr} = \frac{L}{r^2}\left[\sqrt{2\mu\left(E - \frac{L^2}{2\mu r^2} - V(r)\right)}\right]^{-1}$$

- Effective potential:

$$V_{\mathrm{ef},L}(r) \equiv \frac{L^2}{2\mu r^2} + V(r) = E\left(\frac{b}{r}\right)^2 + V(r)$$

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- Turning point: corresponds to the distance of closest approach, i.e:

$$\left.\frac{dr}{dt}\right|_{rmin} = 0 \quad \Rightarrow \quad E = \frac{L^2}{2\mu r_{min}^2} + V(r_{min})$$

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- Trajectory:

$$\phi_A(r) = \int_{r_{rmin}}^{r} du \frac{L}{u^2\sqrt{2\mu\left[E - V_{\mathrm{ef},L}(u)\right]}}$$

Time dependent solution of the scattering problem: Hamiltonian

We need to incorporate explicitly in the Hamiltonian the internal structure of the nucleus being excited (eg. projectile).

$$\boxed{H = T_{\mathbf{r}} + h(\xi) + V(\mathbf{r}, \xi)}$$

- T_R: Kinetic energy for projectile-target relative motion.
- $\{\xi\}$: Internal degrees of freedom of the projectile (depend on the model).
- $h(\xi)$: Internal Hamiltonian of the projectile.

$$\boxed{h(\xi)\phi_n(\xi) = E_n\phi_n(\xi)}$$

- $V(\mathbf{r}, \xi)$: Projectile-target interaction.

Time-dependent solution of the scattering problem

- Projectile-target interaction:

$$V(\mathbf{r},\xi) = V_0(\mathbf{r}) + V_{\mathrm{coup}}(\mathbf{r},\xi)$$

- $V_0(\mathbf{r})$ is used to obtain the classical trajectory $\Rightarrow \mathbf{r}(t)$
- Time-evolution of the projectile wavefunction:

$$i\hbar \frac{d\Psi(\xi,\theta,t)}{dt} = \left[V_{\mathrm{coup}}(\mathbf{r}(\theta,t),\xi) + h(\xi)\right]\Psi(\xi,\theta,t)$$

- Initial condition at $t = -\infty$:

$$|\Psi(-\infty)\rangle = |0\rangle$$

Time-dependent solution of the scattering problem: coupled-channels method

- Expansion of solution in $\{\phi_n(\xi)\}$

$$\Psi(\xi,\theta,t) = \sum_{n=0} c_n(\theta,t) e^{-iE_n t/\hbar} \phi_n(\xi)$$

☞ Initial condition: $c_n(\theta,-\infty) = \delta_{n0}$

- Coupled-channels equations:

$$i\hbar \frac{dc_n(\theta,t)}{dt} = \sum_m e^{-i(E_m-E_n)t/\hbar} V_{nm}(\theta,t) c_m(\theta,t)$$

- (Time-dependent) coupling potentials:

$$V_{nm}(\theta,t) = \int d\xi \phi_n^*(\xi) V_{\rm coup}(\mathbf{r}(\theta,t),\xi)\phi_m(\xi)$$

Excitation probabilities and cross sections

- Excitation probability for $0 \to n$ transition:

$$P_n(\theta) = |c_n(\theta, \infty)|^2$$

- Differential cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{0\to n} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm clas} P_n(\theta)$$

- Conservation of total probability:

$$\sum_n P_n(t) = \sum_n |c_n(t)|^2 = 1$$

First-order perturbative solution

Assume $V_{nm}(\theta, t)$ "small":

- $c_0 \approx 1$
- $c_n \ll 1$ (n>0)

$$
\begin{aligned}
i\hbar \frac{dc_n(\theta,t)}{dt} &= \sum_m e^{-i(E_m-E_n)t/\hbar} V_{nm}(\theta,t) c_m(\theta,t) \\
&\approx e^{-i(E_0-E_n)t/\hbar} V_{n0}(\theta,t) c_0(\theta,t) \\
&\approx e^{-i(E_0-E_n)t/\hbar} V_{n0}(\theta,t)
\end{aligned}
$$

$$
c_n(\theta) \equiv c_n(\theta,\infty) \simeq \frac{1}{i\hbar} \int_{-\infty}^{\infty} e^{-i(E_0-E_n)t/\hbar} V_{n0}(\theta,t) dt
$$

Adiabaticity parameter: adiabatic and sudden limits

- Characteristic excitation time: $\tau_{\rm exc} \sim \hbar/(E_n - E_0)$
- Adiabaticity parameter:

$$\xi_{ad} \equiv \frac{\text{collision time}}{\text{excitation time}} = \frac{\tau_{\rm col}}{\tau_{\rm exc}} = \frac{E_n - E_0}{\hbar}\tau_{col}$$

$$c_n^{(1st)}(\theta) = \frac{1}{i\hbar}\int_{-\infty}^{\infty} \exp\left\{i\frac{E_n - E_0}{\hbar}t\right\} V_{n0}(\theta, t)dt$$

- Limits:
  - $\xi_{ad} \ll 1$ (sudden limit) (fast collision / small excitation energies)
  - $\xi_{ad} \gg 1$ (adiabatic limit) (slow collision / large excitation energies)

    [oscillatory integrand $\Rightarrow P_n(\theta)$ small]

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    [oscillatory integrand $\Rightarrow P_n(\theta)$ small]

☞ *Optimal collision energy to produce maximum excitation:*

$$\xi_{ad} = \frac{(E_n - E_0)}{\hbar}\tau_{col} \approx 1$$

Perturbative solution; higher order effects

- Assume: $V(\mathbf{r},\xi) = V_0(\mathbf{r},\xi) + V_1(\mathbf{r},\xi) \quad (V_1 \ll V_0)$
- Expand solution coefficients as:

$$c_n(t) = c_n^{(0)} + c_n^{(1)} + \dots$$

- Zero-order solution $(V_1 = 0) \Rightarrow c_n^{(0)}$

$$i\hbar \frac{dc_n^{(0)}(\theta,t)}{dt} = \sum_m e^{-i\omega_{nm}t} \langle n|V_0(\theta,t)|m\rangle c_m^{(0)}(\theta,t) \qquad \omega_{nm} \equiv \frac{(E_n - E_m)}{\hbar}$$

- First-order correction $\Rightarrow c_n^{(1)}$

$$i\hbar \frac{dc_n^{(1)}(\theta,t)}{dt} = \sum_m e^{-i\omega_{nm}t} \langle n|V_0(\theta,t)|m\rangle c_m^{(1)}(\theta,t) + \sum_m e^{-i\omega_{nm}t} \langle n|V_1(\theta,t)|m\rangle c_m^{(0)}(\theta,t)$$

Alternative expression for the first-order solution

$$c_n^{(1)}(\infty) = \frac{1}{i\hbar} \sum_{l,m} \int_{-\infty}^{+\infty} \tilde{c}_{l,n}^{(0)}(t') \langle l | V_1(t') | m \rangle c_{m,0}^{(0)}(t') \exp\left\{ \frac{i}{\hbar}(E_l - E_m)t' \right\} dt'$$

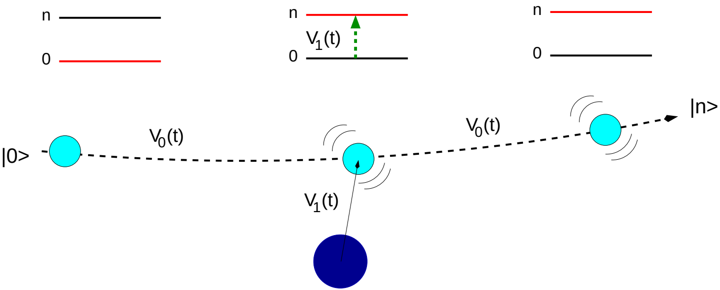
☞ $\tilde{c}_{m,0}^{(0)}(t)$ and $\tilde{c}_{l,n}^{(0)}(t)$ solutions of coupled-equations for V_0 with boundary conditions:

- ⇨ $c_{m,0}^{(0)}(-\infty) = \delta_{0,m}$
- ⇨ $\tilde{c}_{l,n}^{(0)}(+\infty) = \delta_{n,l}$

A simple example: $V_0(\mathbf{r})$ independent of ξ

- $V(\mathbf{r},\xi) = V_0(\mathbf{r}) + V_1(\mathbf{r},\xi)$
- $V_0(\mathbf{r})$ does not produce transitions between the internal states:

$$c_n^{(1)}(\infty) = \frac{1}{i\hbar}\int_{-\infty}^{+\infty} \tilde{c}_n^{(0)}(t')\,\langle n|V_1(t')|0\rangle\, c_0^{(0)}(t')\exp\left\{\frac{i}{\hbar}(E_n - E_0)t'\right\} dt'$$



Application to Coulomb excitation

- Half of distance of closest approach in head-on collision:

$$a_0 = \frac{\kappa Z_p Z_t e^2}{2E}$$

- Distance of closest approach for scattering angle θ:

$$b(\theta) = a_0 \left[1 + \frac{1}{\sin(\theta/2)}\right] \equiv a_0 \left[1 + \epsilon\right]$$

(ϵ=excentricity)

- Adiabaticity parameter for head-on collision:

$$\xi_{0\to n} \equiv \xi_{0\to n}(\theta = \pi) = \frac{(E_n - E_0)}{\hbar}\tau_{col} \approx \frac{(E_n - E_0)}{\hbar}\frac{a_0}{v}$$

- Adiabaticity parameter for general collision

$$\xi_{0\to n}(\theta) = \frac{(E_n - E_0)}{\hbar}\frac{b(\theta)}{2v} = \xi_{0\to n}\left[1 + \frac{1}{\sin(\theta/2)}\right]$$

Validity of the semiclassical approximation

Sommerfeld parameter:

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v} = \frac{a_0}{\lambda\!\!\!^{-}}$$

The projectile is described by a wavepacket of dimension $\sim \lambda\!\!\!^{-}$, which must be small compared to the dimensions of the classical trajectory ($\sim a_0$):

$$\boxed{\lambda\!\!\!^{-} \ll a_0 \;\; \Rightarrow \;\; \eta \gg 1}$$

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$$\lambda\!\!\!^{-} \ll a_0 \quad \Rightarrow \quad \eta \gg 1$$

Also, the trajectory must be barely perturbed by the momentum and energy transfer to the projectile:

$$\frac{\Delta \ell}{\ell} \ll 1 \quad \text{and} \quad \frac{\Delta E_n}{E} \ll 1$$

Multipole expansion of Coulomb potential

- Multipolar expansion of Coulomb potential

$$V(\mathbf{r},\xi) = \frac{Z_t Z_p e^2}{r} + \sum_{\lambda>0,\mu} \frac{4\pi Z_t e}{2\lambda+1} M(E\lambda,\mu) \frac{Y_{\lambda\mu}(\hat{r})}{r^{\lambda+1}}$$

$$\equiv V_0(r) + V_{\text{coup}}(\mathbf{r},\xi)$$

☞ $V_0(r)$ *determines de trajectory, but does not induce excitations*

- Internal states are conveniently labeled by their intrinsic spin and its projection:

$$|n\rangle \rightarrow |I_n M_n\rangle$$

First order Coulomb excitation probability

- $V_0(r)$ determines de trajectory, for each θ.
- In terms of the dimensionless parameter ω:

$$\begin{aligned}
\rho(\omega) = r(\omega)/a_0 &= \epsilon \cosh\omega + 1 \\
\phi(\omega) &= \arctan(\sqrt{\epsilon^2 - 1}\sinh\omega/(\epsilon + \cosh\omega)) \\
\theta(\omega) &= \pi/2 \\
\tau(\omega) = t(\omega)v/a_0 &= \epsilon\sinh\omega + \omega
\end{aligned}$$

First order Coulomb excitation probability

In general:

$$c_n^{(1st)}(\theta) = \frac{1}{i\hbar}\int_{-\infty}^{\infty} e^{i(E_n-E_0)t/\hbar} V_{n0}(\theta,t)dt$$

For the Coulomb case:

$$c_n^{(1st)}(\theta) = \frac{4\pi Z_t e}{i\hbar}\sum_{\lambda\mu}\frac{\langle I_n \| M(E\lambda) \| I_0\rangle}{(2\lambda+1)}\frac{\langle I_n M_n | \lambda\mu I_0 M_0\rangle}{\sqrt{2I_n+1}}\int_{-\infty}^{\infty} dt e^{i(E_n-E_0)t/\hbar}\frac{Y_{\lambda\mu}(\theta,\phi)}{r^{\lambda+1}}$$

$$c_n^{(1st)}(\theta) = \frac{4\pi Z e^2}{i\hbar v}\sum_{\lambda}\frac{\langle I_n \| M(E\lambda) \| I_0\rangle}{(2\lambda+1)e a_0^{\lambda}}\frac{\langle I_n M_n | \lambda\mu I_0 M_0\rangle}{\sqrt{2I_n+1}} I_{\lambda\mu}(\epsilon,\xi) Y_{\lambda\mu}(\pi/2,0)$$

☞ Coulomb integrals:

$$I_{\lambda\mu}(\epsilon,\xi) = \int_{-\infty}^{\infty} d\omega \rho(\omega)^{-\lambda} \exp[i\xi\tau(\omega)] \exp[i\mu\phi(\omega)]$$

First order Coulomb excitation probability

- Excitation probability:

$$P_n = \left(\frac{4\pi Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, g \to n)}{(2\lambda+1)^3 e^2 a_0^{2\lambda}} \sum_\mu \left(Y_{\lambda\mu}(\pi/2) I_{\lambda,\mu}(\epsilon,\xi)\right)^2$$

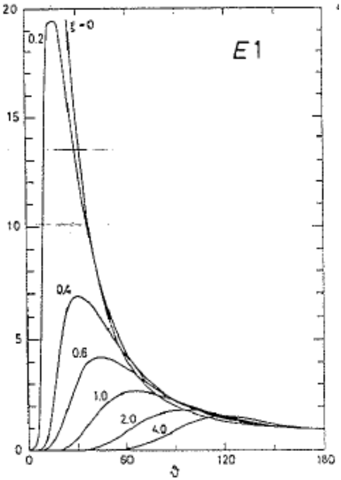
- Differential cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{0\to n} = \left(\frac{Ze^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0 \to n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta,\xi)$$

$$f_\lambda(\theta,\xi) = \frac{4\pi^2}{(2\lambda+1)^3 \sin^4(\theta/2)} \sum_\mu \left[Y_{\lambda\mu}(\pi/2,0) I_{\lambda\mu}(\epsilon,\xi)\right]^2$$

Coulomb integrals

$f_\lambda(\theta, \xi)$ for $\lambda = 1$ (E1) transitions:



- $f_\lambda(\theta, \xi)$ small for large ξ (adiabatic limit)
- For $\theta \to 0 \Rightarrow \xi_{0\to n}(\theta) \to \infty \Rightarrow P_n(\theta) \to 0$

Application to Coulomb dissociation of halo nuclei

- For excitation to bound states ($0 \rightarrow n$)

$$\left(\frac{d\sigma}{d\Omega}\right)_{0\rightarrow n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0 \rightarrow n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta, \xi)$$

- Halo nuclei are weakly bound $\Rightarrow$ excitation occurs to unbound (continuum) states

$$\boxed{\frac{d\sigma(E\lambda)}{d\Omega dE} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{dE} \frac{df_\lambda(\theta, \xi)}{d\Omega}}$$

Photo-absorption cross section: virtual photon description

➩ Photo-absorption (not proven here): $\gamma + a \longrightarrow b + c$

$$\sigma^{\rm photo}_{E\lambda} = \frac{(2\pi)^3(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(E\lambda)}{dE}$$

➩ 1^{st} order Coulomb breakup cross section in terms of photo-absorption:

$$\frac{d\sigma(E\lambda)}{d\Omega dE_\gamma} = \frac{1}{E_\gamma}\frac{dn_{E\lambda}}{d\Omega}\sigma^{\rm photo}_{E\lambda}$$ (Equivalent Photon Method)

with the virtual photon number

$$\frac{dn_{E\lambda}}{d\Omega} = Z_t^2\alpha\frac{\lambda[(2\lambda+1)!!]^2}{(2\pi)^3(\lambda+1)}\xi^{2(1-\lambda)}\left(\frac{c}{v}\right)^{2\lambda}\frac{df_{E\lambda}}{d\Omega}$$

Extracting dB/dE from CD experiments

- Assume that Coulomb breakup is dominated by E1:

$$\sigma_{E1}^{(\mathrm{photo})} = \frac{16\pi^3}{9}\frac{E_\gamma}{\hbar c}\frac{dB(E1)}{dE_\gamma} \quad \Rightarrow \quad \boxed{\frac{d^2\sigma}{d\Omega dE_x} = \frac{16\pi^3}{9}\frac{dn_{E1}}{d\Omega}\frac{dB(E1)}{dE_x}}$$

- Assume that CD is Coulomb for $b > b_0$ $(\theta < \theta_{gr})$

$$\frac{d\sigma}{dE_x}(\theta < \theta_{gr}) = \int_0^{\theta_{gr}} d\Omega \frac{dn_{E1}}{d\Omega}\frac{\sigma_{E\lambda}}{E_x} \equiv n_{E\lambda}(E_x)\frac{\sigma_{E\lambda}}{E_x}$$

$$\boxed{\frac{d\sigma}{dE_x}(\theta < \theta_{gr}) = \frac{16\pi^3}{9} n_{E\lambda}(E_x)\frac{dB(E1)}{dE_x}}$$

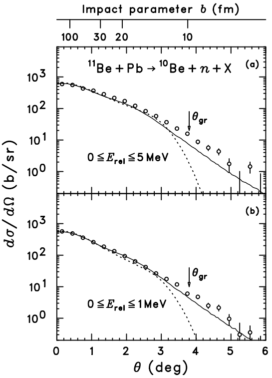
Application a CD of 1n-halo nuclei

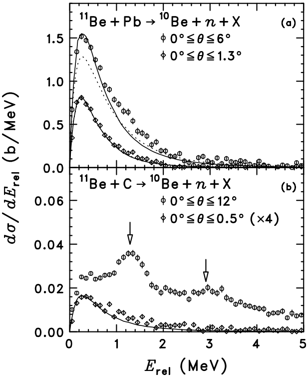
- Halo nuclei are weakly bound ⇒ the systems are easily polarized in strong electric field (large E1 response)
- Large Coulomb dissociation probability with heavy targets
- At small-angles (large impact parameters) the dissociation is Coulomb dominated and hence can be used to extract the $E1$ transition probability using:

$$\frac{d\sigma}{dE_x}(\theta < \theta_{gr}) = \frac{16\pi^3}{9} n_{E\lambda}(E_x) \frac{dB(E1)}{dE_x}$$

Example: ^{11}Be dissociation

$$^{11}\text{Be}+^{208}\text{Pb} \rightarrow ^{10}\text{Be}+\text{n}+^{208}\text{Pb} \ @ \ 69 \text{ MeV/u}$$

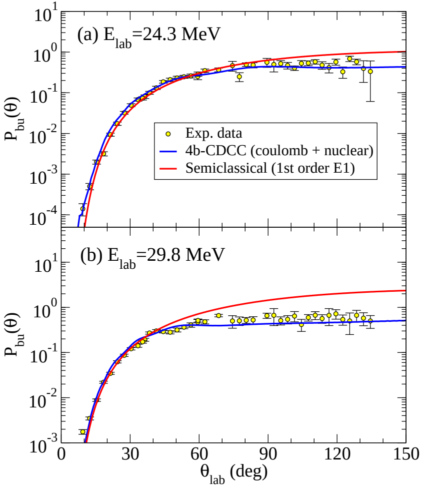




Fukuda, 2004

Comparison of 1st order with full quantum-mechanical calculation

Eg: ^{11}Li+^{208}Pb at Coulomb barrier energies



- $E_{lab} \sim V_b \Rightarrow$ Coulomb important
- At small angles, breakup dominated by E1 Coulomb

J.P. Fernandez-Garcia et al, PRL110, 142701(2013)

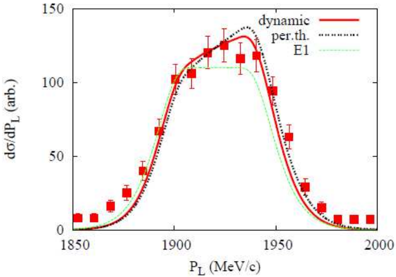
Higher-order corrections

Deviations from 1st order $E1$ perturbation theory:

- Higher multipoles (λ>1)
- Higher orders in a expansion of c_n amplitudes

$$c_{if} = c_{if}^{(1)} + c_{if}^{(2)} + \ldots$$

Eg.: ^{8}B+^{197}Au → ^{7}Be+p+^{197}Au @ 40 MeV/u

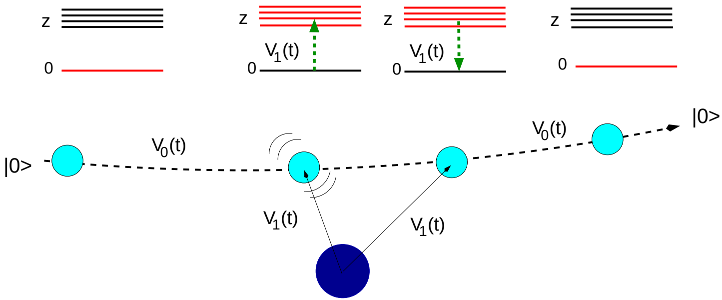


⇨ Asymmetry due to E1/E2 interference reduce effect

Esbensen, NPA600, 37 ('96); *Data: Kelley, PRL77, 5020 ('96)*

Second order perturbative amplitude

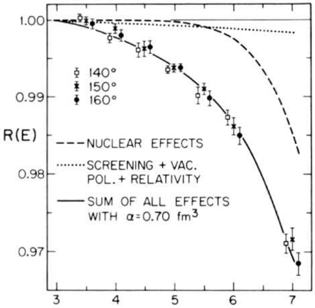
$$c_n^{(2)} = \sum_z \left(\frac{-i}{\hbar}\right)^2 \int_{-\infty}^{+\infty} dt \langle n|V_1(t)|z\rangle \exp\left\{\frac{i}{\hbar}(E_n - E_z)t\right\}$$
$$\times \int_{-\infty}^{t} dt' \langle z|V_1(t')|0\rangle \exp\left\{\frac{i}{\hbar}(E_z - E_0)t'\right\}$$



The effect of the inelastic couplings on the elastic scattering

Eg: deuteron polarizability from d+^{208}Pb:

- ☞ Deuteron polarizability: $\mathbf{P} = \alpha \mathbf{E}$
- ☞ For $E < V_b$, the main deviation from Rutherford scattering comes from dipole polarizability.
- ☞ The second order correction can be included in an effective (polarization) potential. In the adiabatic limit: $V_{\rm dip} = -\alpha \frac{Z_1 Z_2 e^2}{2R^4}$



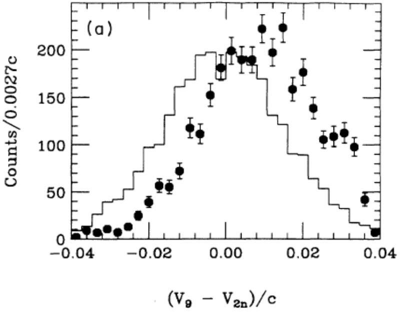
Rodning, Knutson, Lynch and Tsang,

PRL49, 909 (1982)

$\alpha = 0.70 \pm 0.05 \text{ fm}^3$

Post-acceleration effects

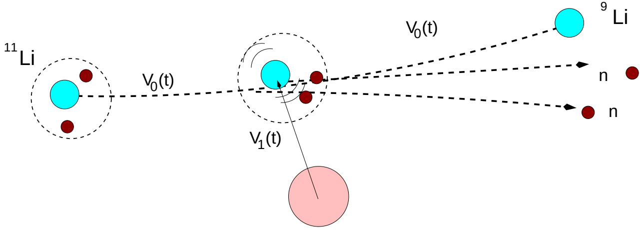
^{11}Li+Pb $\rightarrow$ ^{9}Li+n+n+Pb @ 28 MeV/u



Sacket, PRC48, 118 (1993)

- After breakup, ^{9}Li accelerated due to Coulomb field
- Not explained by first order model.

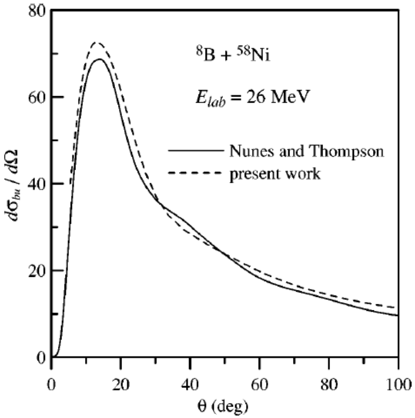
Post-acceleration effect (cont.)



- The projectile approaches as a whole, and breaks up at a given distance from the target (R_{bu})
- At this energy, the potential energy is $V(R_{bu}) \approx Z_p Z_t e^2/R_{bu}$
- After breaking up, this energy is transformed into kinetic energy, but only of ^{9}Li.

$$\Delta E(^9\text{Li}) \approx Z_p Z_t e^2/R_{bu}$$

Coupled-channels calculations with Coulomb+nuclear



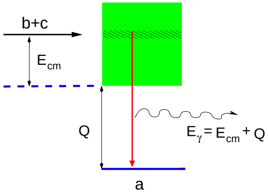
H.D. Marta, PRC66, 024605 ('02)

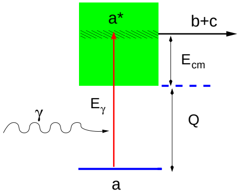
- Full coupled-channels (all orders)
- Coulomb+nuclear trajectory
- Several multipoles

Relation to radiative capture

Radiative capture: $b + c \to a + \gamma$

Photo-absorption: $a + \gamma \to b + c$





➪ Related by detailed balance:

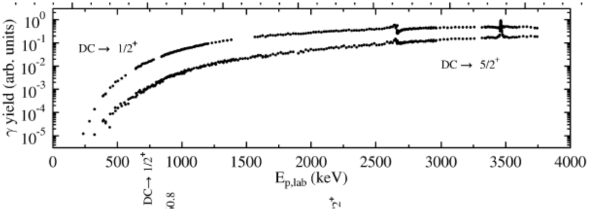
$$\sigma^{(rc)}_{E\lambda} = \frac{2(2J_a+1)}{(2J_b+1)(2J_c+1)} \frac{k_\gamma^2}{k^2} \sigma^{(phot)}_{E\lambda} \qquad (\hbar k_\gamma = E_\gamma/c)$$

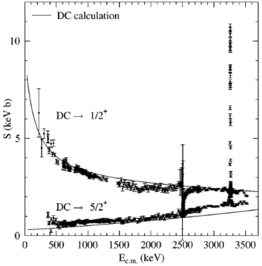
➪ Astrophysical S-factor:

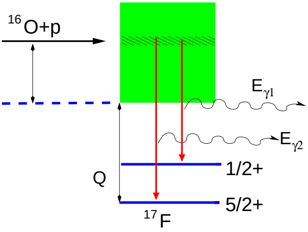
$$S(E_{\mathrm{c.m.}}) = E_{\mathrm{c.m.}} \sigma^{(rc)}_{E\lambda} \exp[2\pi\eta(E_{\mathrm{c.m.}})]$$

Example: p+^{16}O → ^{17}F + γ

Morlock, PRL79, 3837 (1997)







Radiative capture from Coulomb dissociation experiments

- Capture reactions have typically small cross sections
- Use breakup (Coulomb dissociation) reactions:

$$\frac{d\sigma}{d\Omega dE_{c.m.}} \rightarrow \sigma_{E\lambda}^{(\mathrm{phot})} \rightarrow \sigma_{E\lambda}^{(rc)} \rightarrow S(E_{\mathrm{c.m.}})$$