Radiative-capture reactions

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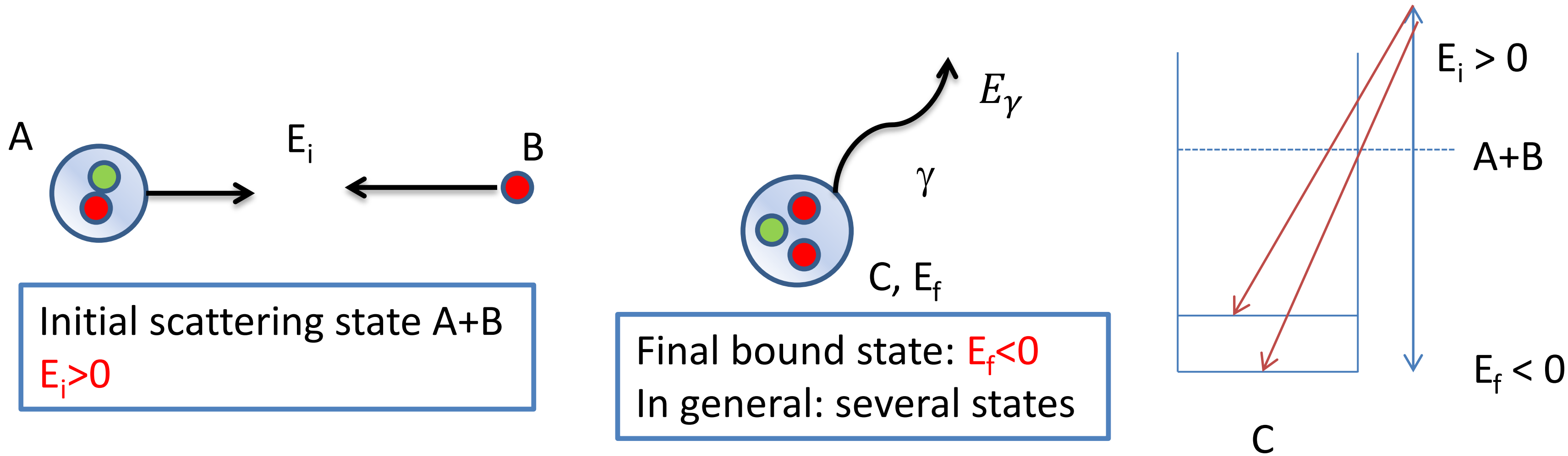
1. Introduction, definitions
2. Electromagnetic transitions
3. Radiative-capture cross sections
4. Radiative capture in nuclear astrophysics
5. Application to the potential model
6. Conclusions

Capture reaction = Electromagnetic transition from a scattering state to a bound state



Energy conservation: $E_\gamma = E_i - E_f$ (recoil energy of nucleus C is negligible)

Examples : (p,γ) reactions: ^{7}Be(p,γ)^{8}B, ^{12}C(p,γ)^{13}N, ^{16}O(p,γ)^{17}F
(α,γ) reactions: ^{3}He(α,γ)^{7}Be, ^{12}C(α,γ)^{16}O

Typical values: up to a few MeV

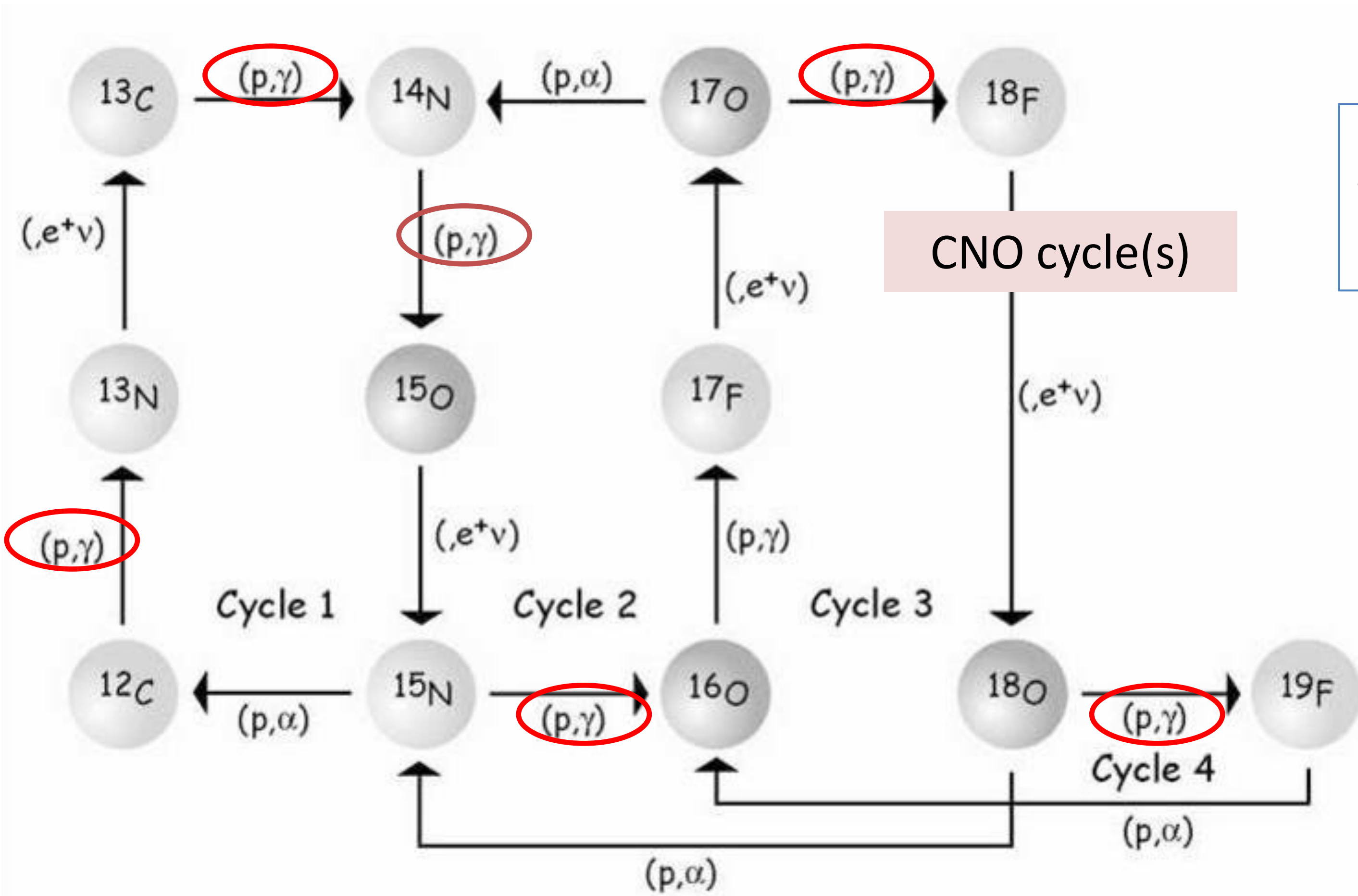
Photon wavelength for $E_\gamma = 1$ MeV: $\lambda_\gamma = \frac{2\pi}{k_\gamma} = \frac{2\pi\hbar c}{E_\gamma} \sim 1200$ fm

$\lambda_\gamma \gg$ typical dimension R

$k_\gamma R \ll 1$: long wavelength approximation

Importance in astrophysics:

1. **Hydrogen burning: CNO cycle, pp chain**



Capture reactions
Transfer reactions
Beta decays

2. **Helium burning:** $^{8}Be(\alpha,\gamma)^{12}C$, $^{12}C(\alpha,\gamma)^{16}O$, $^{16}O(\alpha,\gamma)^{20}Ne$,…

3. Neutron capture reactions: $^{7}Li(n,\gamma)^{8}Li$, many others…

Difference fusion / radiative capture

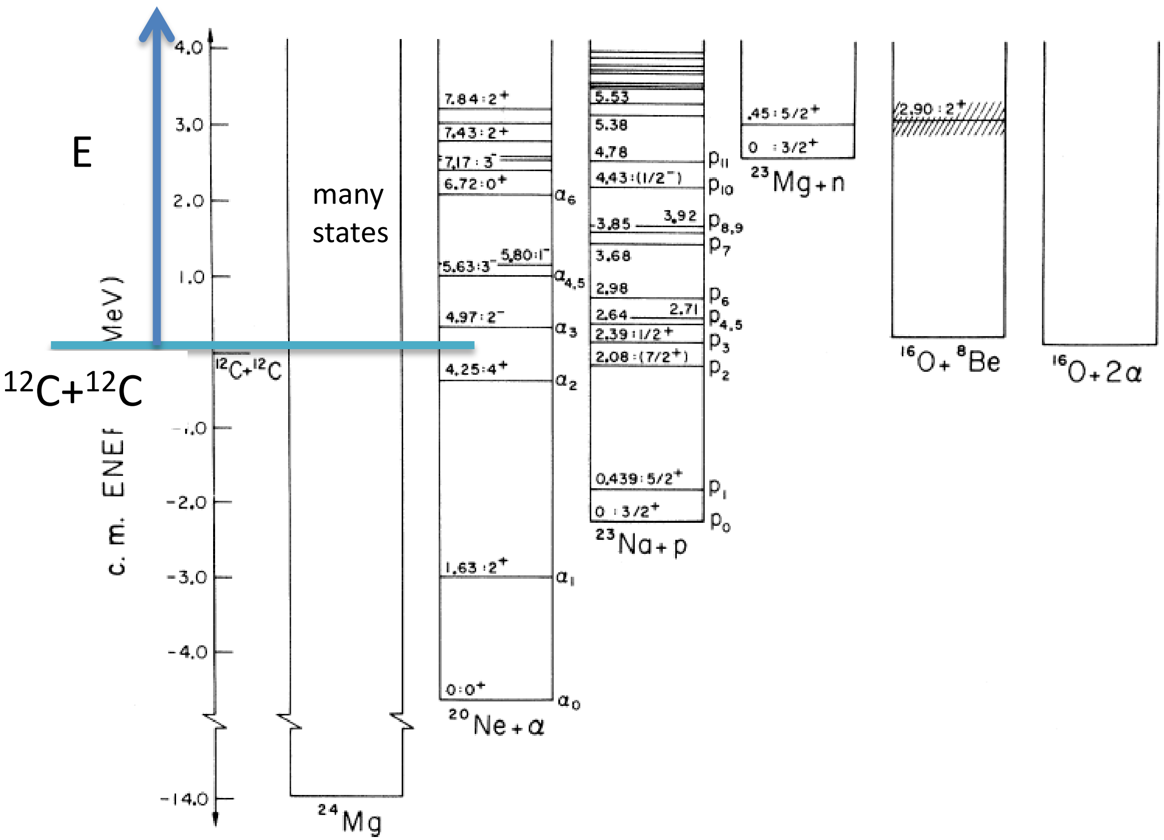
Radiative capture: γ channel only (electromagnetic)

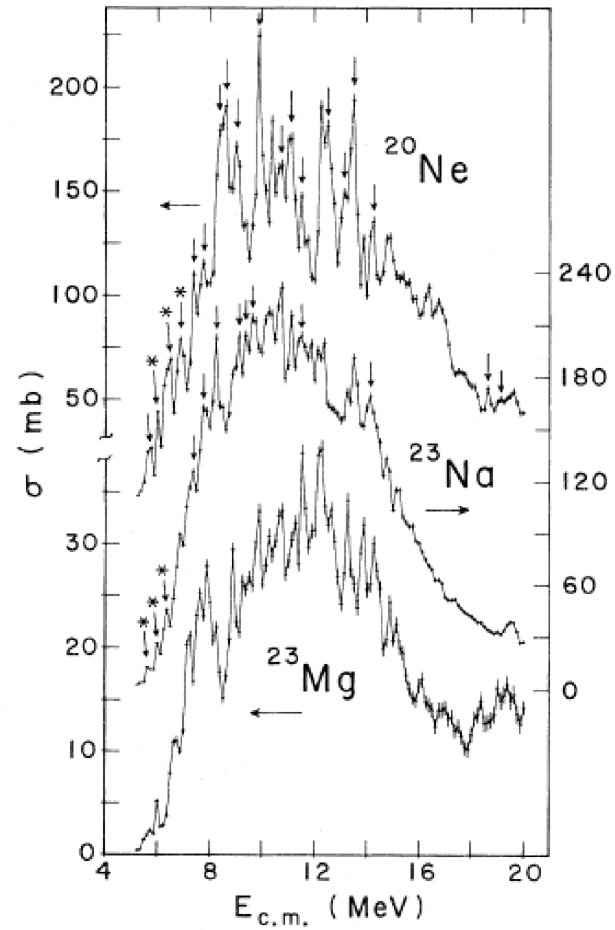
Fusion:

- all channels with mass higher than the projectile and target
- In general, many channels → statistical approach

experimental cross section
Satkowiak et al. PRC 26 (1982) 2027

Example: $^{12}C+^{12}C$ fusion



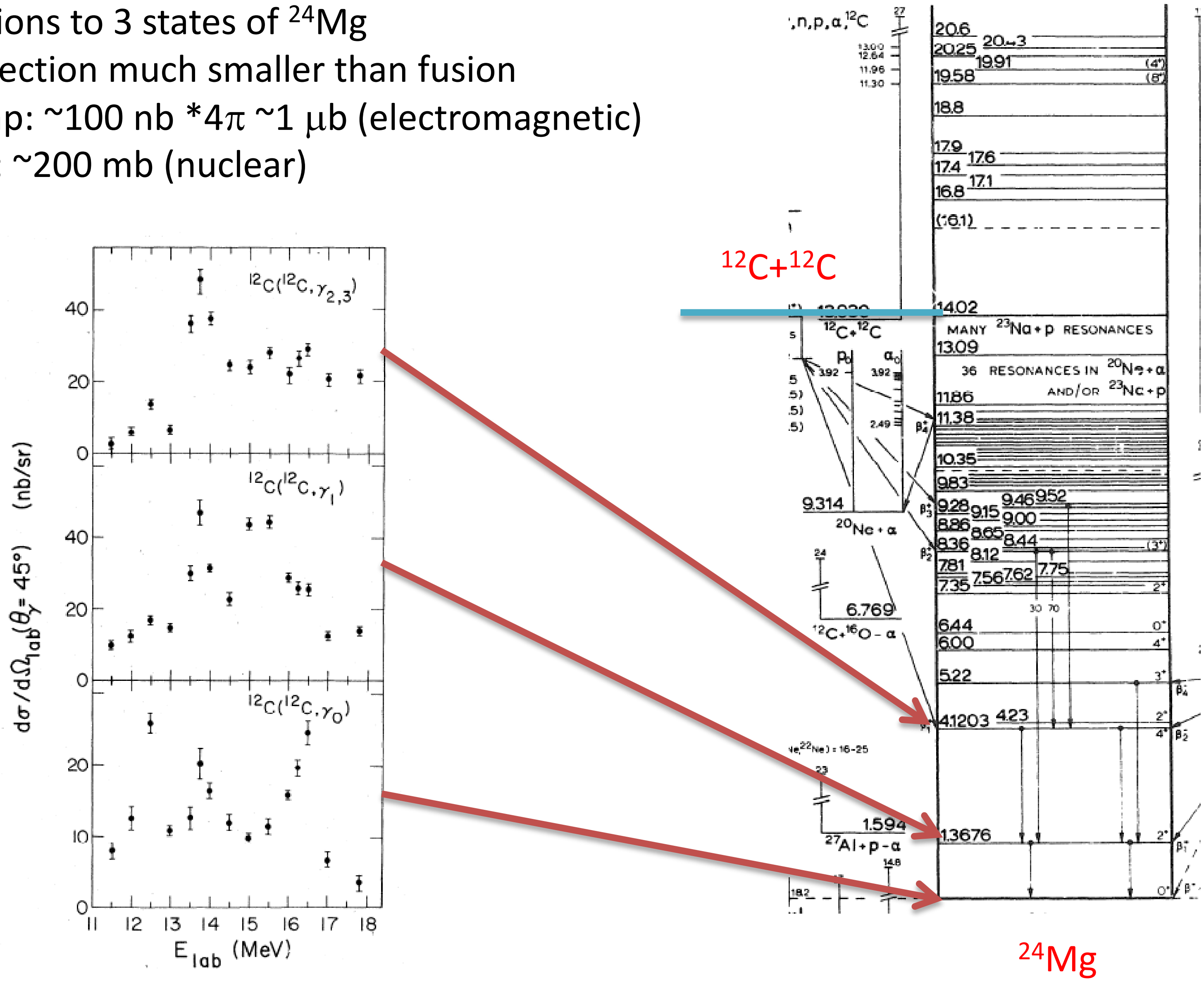


Other example: $^{12}C(^{12}C,\gamma)^{24}Mg$ radiative capture

- Existence of molecular states in ^{24}Mg?
- Transitions to 3 states of ^{24}Mg
- Cross section much smaller than fusion

  Rad. cap: ~100 nb *4π ~1 μb (electromagnetic)

  Fusion: ~200 mb (nuclear)



2. Electromagnetic transitions

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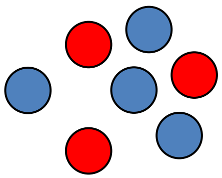
Transition probability

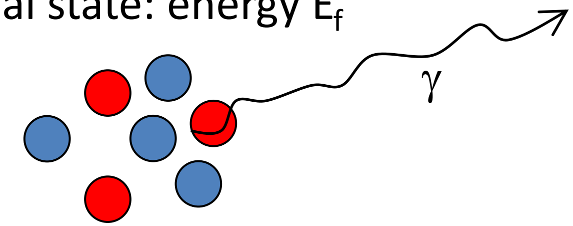
Initial state: energy E_i



system of charged particles:
$H = H_0$

Wave function Ψ_i

Final state: energy E_f



system of charged particles + photon:
$H = H_0 + H_\gamma$ (H_γ small)

Wave function Ψ_f

- Energy conservation: $E_i = E_f + E_\gamma$
- Perturbation theory (H_γ small)
- Fermi golden rule

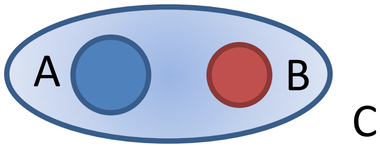
→ Transition probability from an initial state to a final state

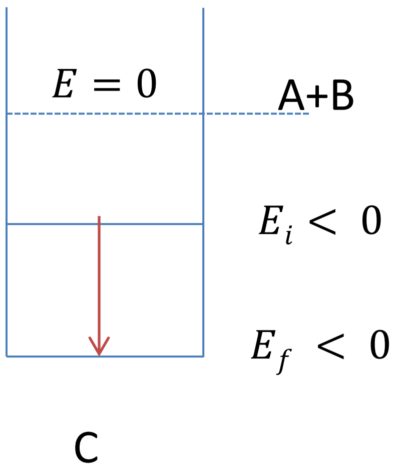
→ $w_{i \to f} \sim |< \Psi_f | H_\gamma | \Psi_i >|^2$: very general definition

2. Electromagnetic transitions

Three types of electromagnetic transitions

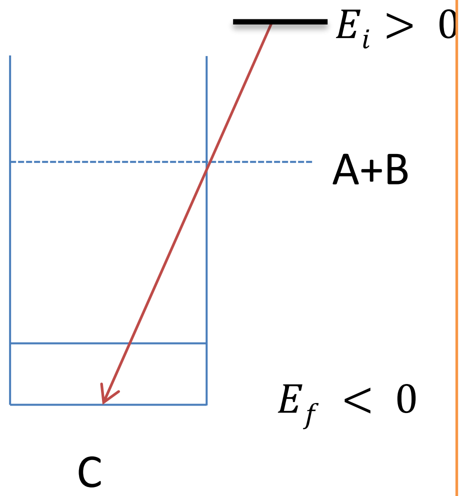
Nucleus C=A+B





Bound → bound

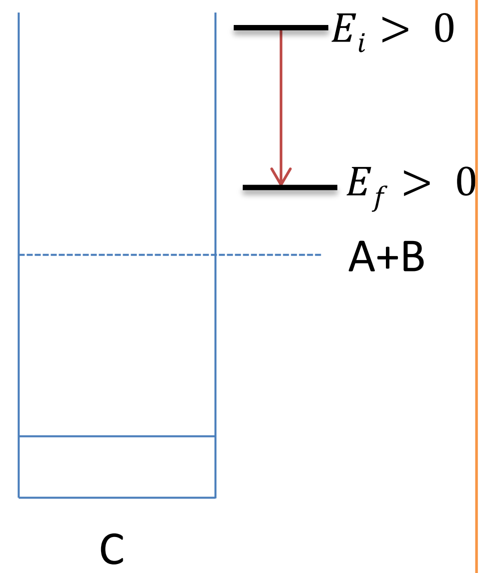
$E_i < 0, E_f < 0$



scattering → bound

$E_i > 0, E_f < 0$

=radiative capture



scattering → scattering

$E_i > 0, E_f > 0$

=bremsstrahlung

- In all cases $w_{i \to f} \sim \left| < \Psi_f \middle| H_\gamma \middle| \Psi_i > \right|^2$
- But: different types of wave functions
- Bremsstralhung: transition from continuum to continuum → strong convergence problems

Bound→ bound:

- Initial and final states characterized by spins $J_i\pi_i$ and $J_f\pi_f$
- Transition probability: $w_{i\to f} \sim |< \Psi^{J_f\pi_f}|H_\gamma|\Psi^{J_i\pi_i} > |^2$
- $w_{i\to f}$ provides the γ width $\Gamma_\gamma = \hbar w$, lifetime $\tau = \hbar/\Gamma_\gamma$

Scattering to bound state: radiative capture

- Final state: characterized by spin $J_f\pi_f$
- Initial state : $\Psi(E)$ = scattering state (expansion in partial waves, all $J_i\pi_i$)
- Transition probability: $w_{i\to f}(E) \sim |< \Psi^{J_f\pi_f}|H_\gamma|\Psi(E) > |^2$
- $w_{i\to f}$ provides the capture cross section

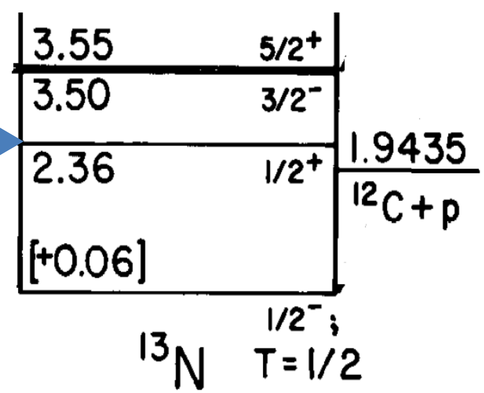
General property : H_γ small → electromagnetic processes have a low probability

- Gamma widths small compared to particle widths
- Capture cross sections small compared to elastic or transfer cross sections (nuclear origin)

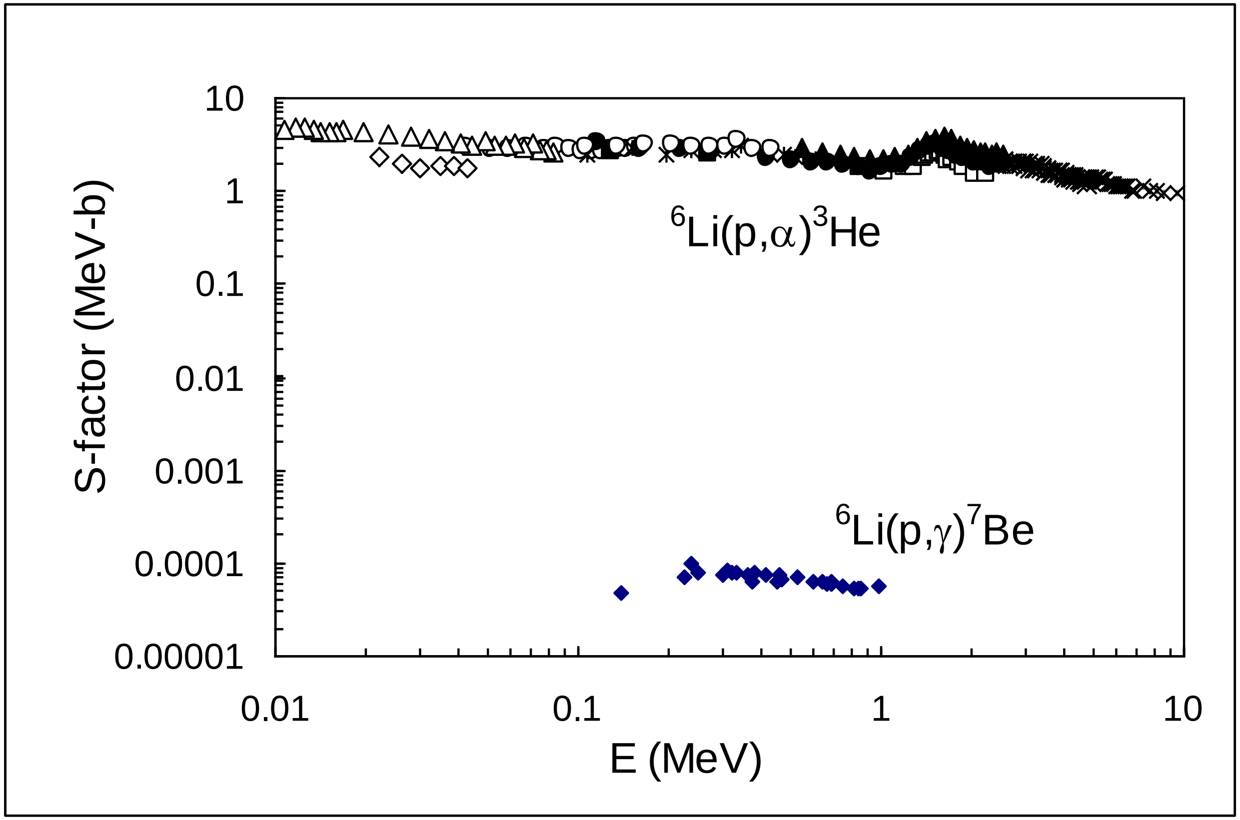
γ decay: Γ_γ**=0.5 eV,** T=10^{-14} s

proton decay: Γ_p**=32 keV,** T=2×10^{-20} s

→Γ_γ/Γ_p**<<1**



Comparaison transfer – capture reactions: $^6Li(p,\alpha)^3He$ and $^6Li(p,\gamma)^7Be$



S- Factor

$$S(E) = \sigma(E) E \exp(2\pi\eta)$$

- factor 10^4 typical of nucl/elec.
- γ channel negligible compared to α channel

S factor proportional to σ

Same energy dependence (identical entrance channel)

σ_t (transfer)>> σ_C (capture) : general property

2. Electromagnetic transitions

Electromagnetic operator H_γ

- Depends on
  - nuclear coordinates (nuclei or nucleons, according to the model)
  - photon properties (energy E_γ, emission angle Ω_γ)

- Deduced from the Maxwell equations (+quantification)

$$H_\gamma \sim \sum_{\lambda\mu\sigma} k_\gamma^\lambda \mathcal{M}_\mu^{\sigma\lambda}(r_1, \dots r_A) D_{\mu q}^\lambda(\Omega_\gamma)$$

nucleons → $\mathcal{M}_\mu^{\sigma\lambda}(r_1, \dots r_A)$

photon → $D_{\mu q}^\lambda(\Omega_\gamma)$

  - $\sigma = E$ or M (electric or magnetic)
  - λ=order of the multipole (from 1 to ∞, in practice essentially $\lambda = 1,2$)
  - μ is between $-\lambda$ and $+\lambda$
  - $D_{\mu q}^\lambda(\Omega_\gamma)$=Wigner fonction , Ω_γ=photon-emission angle
  - $\mathcal{M}_\mu^{\sigma\lambda}$ = multipole operators (depend on nucleon coordinates $\boldsymbol{r}_i$)

- **Electric operators**

$\widetilde{\mathcal{M}}_{\mu}^{E\lambda} = e \sum_i (\frac{1}{2} - t_{iz}) r_i^{\lambda} Y_{\lambda}^{\mu}(\Omega_{ri})$ for a many-nucleon system (sensitive to protons only)

with t_{iz}=isospin= +1/2 (neutron)
-1/2 (proton)

$\boldsymbol{r_i} = (r_i, \Omega_i)$ = nucleon space coordinate

- **Magnetic operators**

$$\mathcal{M}_{\mu}^{M\lambda} = \frac{\mu_N}{\hbar} \sum_i [\boldsymbol{\nabla}(r^{\lambda} Y_{\lambda}^{\mu}(\Omega_r)]_{r=r_i} \cdot (\frac{2g_{li}}{\lambda+1} \boldsymbol{L}_i + g_{si} \boldsymbol{S}_i)$$

with $\boldsymbol{S}_i$=spin of nucleon i
$\boldsymbol{L}_i$=orbital momentum of nucleon i

- Transition probability (integral over Ω_γ)

$$w_{i\rightarrow f}(E) \sim \left| < \Psi^{J_f \pi_f} \middle| H_\gamma \middle| \Psi^{J_i \pi_i} > \right|^2$$

- H_γ is expanded in multipoles

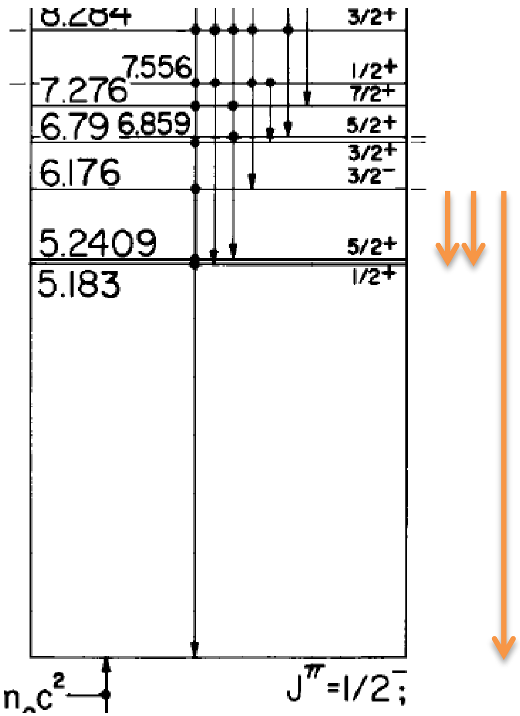
$$W_{J_i\pi_i \rightarrow J_f\pi_f} \sim \sum_{\lambda\sigma} k_\gamma^{2\lambda+1} \left| < \Psi^{J_f\pi_f} \parallel \mathcal{M}^{\sigma\lambda} \parallel \Psi^{J_i\pi_i} > \right|^2$$

photon — nucleus

- Reduced transition probability

$$B(\sigma\lambda, J_i\pi_i \rightarrow J_f\pi_f) = \frac{2J_f+1}{2J_i+1} \left|< \Psi^{J_f\pi_f} \parallel \mathcal{M}^{\sigma\lambda} \parallel \Psi^{J_i\pi_i} >\right|^2$$

- Units: electric (σ=E): $e^2\mathrm{fm}^{2\lambda}$
  magnetic (σ=M): $\mu_N^2\mathrm{fm}^{2\lambda-2}$
- Gamma width: $\Gamma_\gamma(J_i\pi_i \rightarrow J_f\pi_f) = \hbar W_{J_i\pi_i \rightarrow J_f\pi_f}$
- Total gamma width : sum over final states



example: ^{15}O
- from a given state:
  several transitions are possible
- in practice:
  - selection rules
  - factor $k_\gamma^{2\lambda+1}$ favors large E_γ

Important properties:

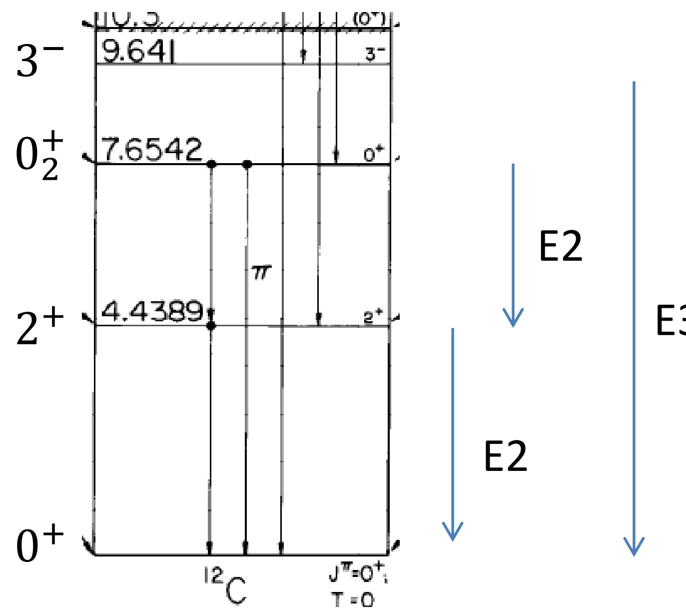
- Hierarchy between the multipoles: $\frac{w(\sigma,\lambda+1)}{w(\sigma,\lambda)} \sim (k_\gamma R)^2 \ll 1$ (long wavelength approximation)
  $E1 \gg E2 \approx M1 \gg E3 \approx M2, \ldots$
  ➔ only a few multipoles contribute (one)

- Selection rules: $< \Psi^{J_f \pi_f} \parallel \mathcal{M}^{\sigma\lambda} \parallel \Psi^{J_i \pi_i} >$
  angular momentum $|J_i - J_f| \leq \lambda \leq J_i + J_f$
  parity: $\pi_i \pi_f = (-1)^\lambda$ for $\sigma = E$
  $\pi_i \pi_f = (-1)^{\lambda+1}$ for $\sigma = M$

- E1 forbidden in N=Z nuclei (T=0)
  $$M^{E1} \sim \sum(\tfrac{1}{2} - t_{iz})(\boldsymbol{r_i} - \boldsymbol{R}_{cm})$$

- No transition with $\lambda = 0$

- Examples:
  - transition $2^+ \rightarrow 0^+ : E2$
  - transition $1^- \rightarrow 0^+ : E1$
  - transition $2^+ \rightarrow 1^+ : E2, M1, M3$
  - transition $3^- \rightarrow 2^+ : E1, E3, E5, M2, M4$



Capture cross section (for a given final state J_f):

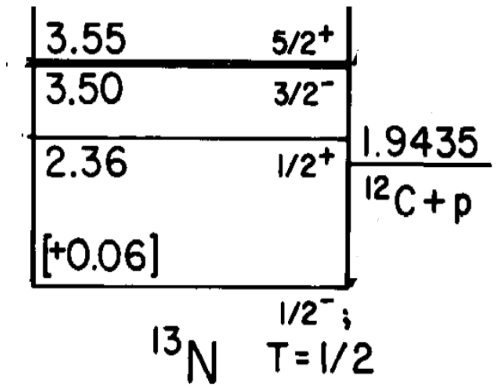
$$\sigma(J_f, E) \sim \sum_{\lambda\sigma} k_\gamma^{2\lambda+1} \left| < \Psi^{J_f \pi_f} \parallel \mathcal{M}^{\sigma\lambda} \parallel \Psi(E) > \right|^2$$

with $\Psi(E)$ =initial state, expanded in partial waves $\Psi(E) = \sum_{J_i} \Psi^{J_i \pi_i}(E)$

E1 (or E2) dominant

→ picks up a few partial waves

examples: $^{12}C(p,\gamma)^{13}N$, $^{12}C(\alpha,\gamma)^{16}O$



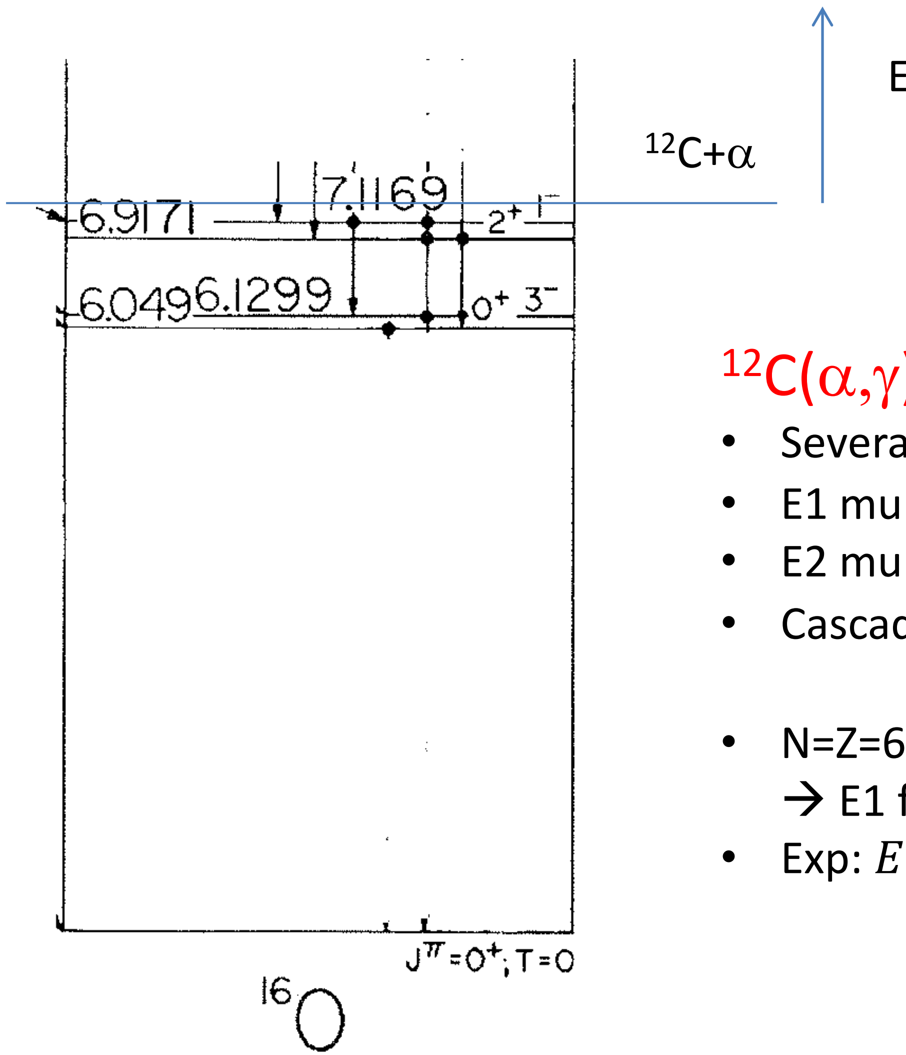
$^{12}C(p,\gamma)^{13}N$

A single final state: $J_f = 1/2^-$ (ground state)

Main multipolarity: E1

→ Initial states: $J_i = 1/2^+, 3/2^+$

Resonance for $J_i = 1/2^+$ → enhances the cross section



$^{12}C(\alpha,\gamma)^{16}O$

- Several final states: ($J_f = 0^+$dominant)
- E1 multipolarity: $J_i = 1^-$
- E2 multipolarity: $J_i = 2^+$
- Cascade transitions small (factor $k_\gamma^{2\lambda+1}$)

- N=Z=6
  → E1 forbidden (T=0, long wavelength approx.)
- Exp: $E1 \approx E2$ (T=0 not strict)

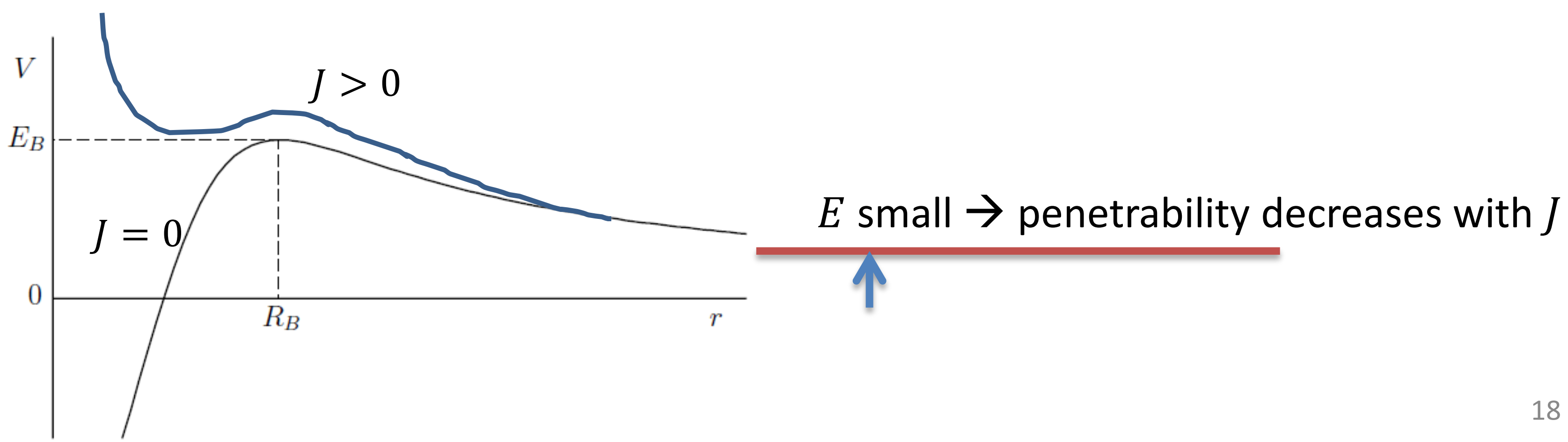
3. Cross sections for radiative capture

We assume a spin zero for the colliding nuclei

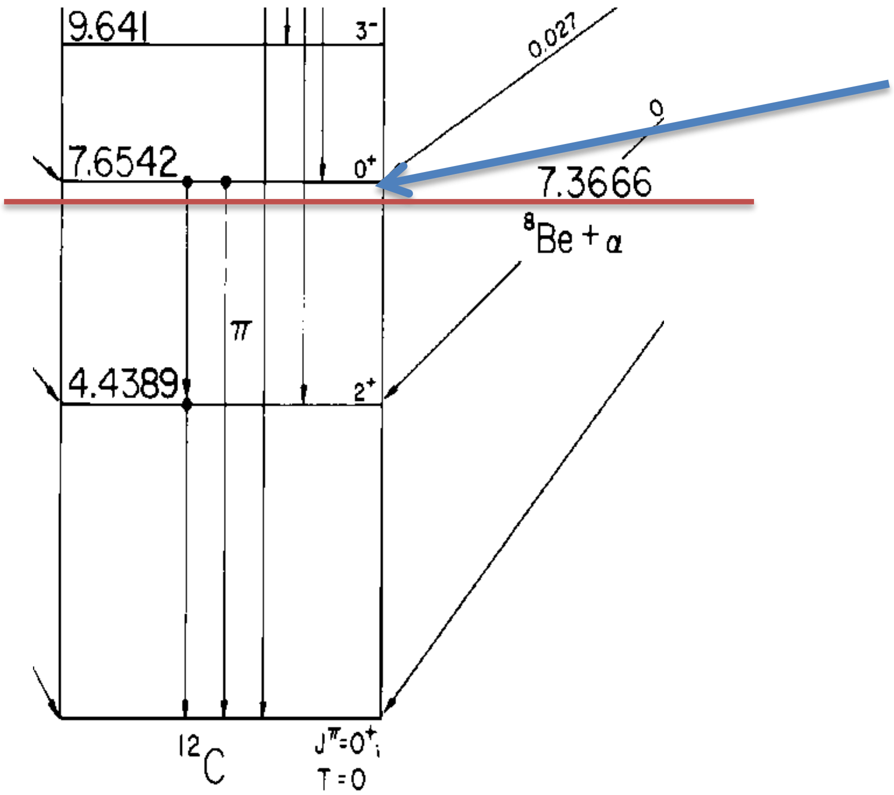
$$\sigma(J_f, E) \sim \sum_{\lambda\sigma} k_\gamma^{2\lambda+1} \left| < \Psi^{J_f \pi_f} \parallel \mathcal{M}^{\sigma\lambda} \parallel \Psi(E) > \right|^2$$

- General definition, valid for any model.
  - Potential model (structure neglected)
  - Microscopic models
  - R-matrix, etc.

- Spins zero for the colliding nuclei: $\Psi(E) = \sum_J \Psi^J(E)$
- Essentially astrophysics applications → low energies → low J values
- At low scattering energies $J_i = 0^+$ is dominant (centrifugal barrier)
  → $J_f = 1^-$ $(E1)$, $J_f = 2^+$ $(E2)$ are expected to be dominant

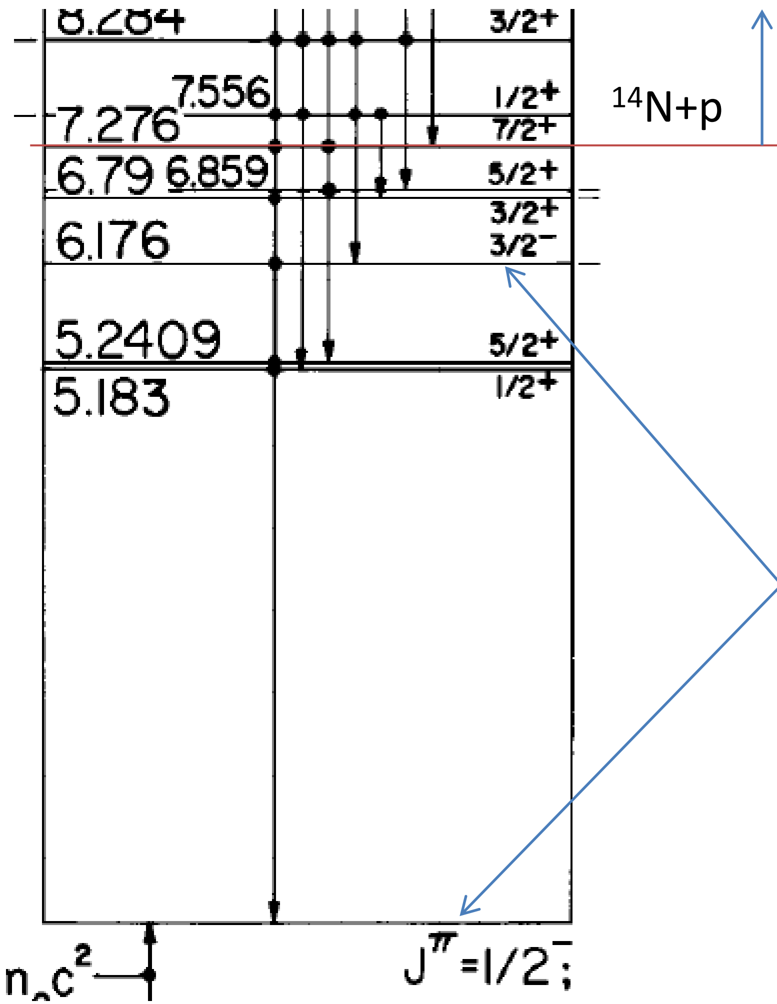


Example 1: $^{8}Be(\alpha,\gamma)^{12}C$



- Initial partial wave $J_i = 0^+$ (includes the Hoyle state.
- E2 dominant (E1 forbidden in N=Z)
- → essentially the $J_f = 2^+$ state is populated.

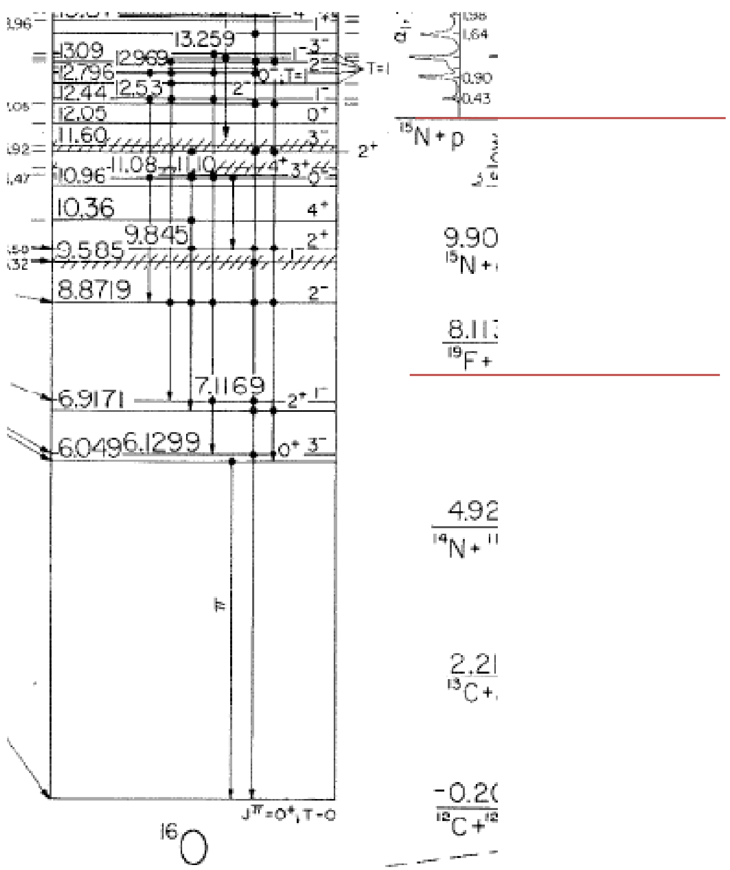
Example 2: $^{14}N(p,\gamma)^{15}O$



- Spin of ^{14}N: I_1=1$^+$, proton I_2=1/2$^+$
- Channel spin I:
  $$|I_1 - I_2| \leq I \leq I_1 + I_2$$
  → $I = 1/2, 3/2$
- Orbital momentum ℓ
  $$|I - \ell| \leq J_i \leq I + \ell$$
- At low energies, $\ell = 0$ is dominant → $J_i = 1/2^+, 3/2^+$
- multipolarity E1 → transitions to $J_f = 1/2^-, 3/2^-, 5/2^-$
- Resonance $1/2^+$ determines the cross section

Radiative capture in astrophysics

- Elastic scattering is always possible, but does not affect the nucleosynthesis
- Essentially two types of reactions
  - Transfer
  - Capture
- capture is important only if transfer channels are closed



^{15}N+p threshold

$^{15}N(p,\gamma)^{16}O$ and $^{15}N(p,\alpha)^{12}C$ are open

→ $^{15}N(p,\gamma)^{16}O$ negligible

$^{12}C+\alpha$ threshold

only possibility: $^{12}C(\alpha,\gamma)^{16}O$

→ $^{12}C(\alpha,\gamma)^{16}O$ (very) important

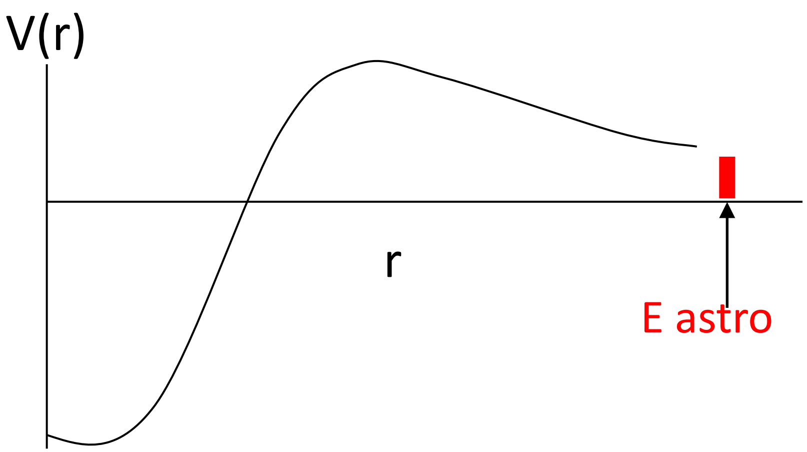
Low energies →

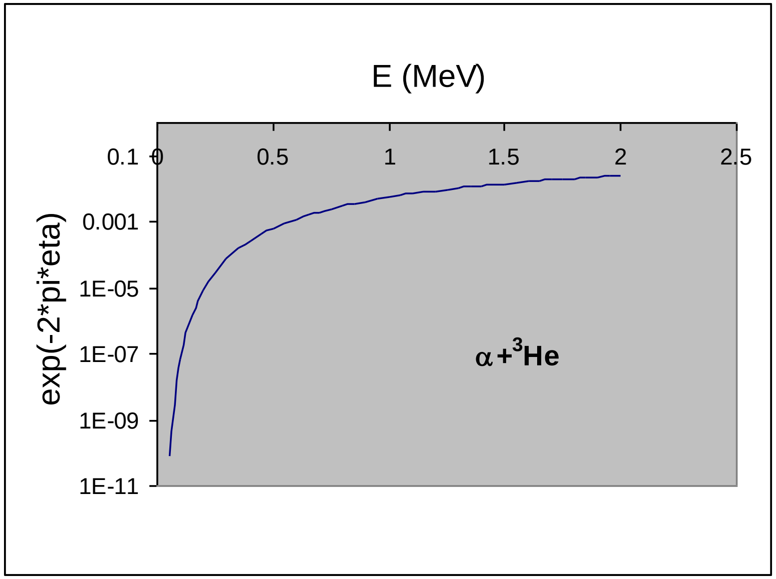
- cross sections dominated by coulomb effects
- Coulomb functions at low energies (η=Sommerfeld parameter=$Z_1Z_2e^2/\hbar v$)
  $F(\eta,x) \rightarrow \exp(-\pi\eta)\mathcal{F}(x)$, $G(\eta,x) \rightarrow \exp(\pi\eta)\mathcal{G}(x)$

➔ 2 coulomb effects:

strong E dependence : factor $\exp(-2\pi\eta)$

strong ℓ dependence



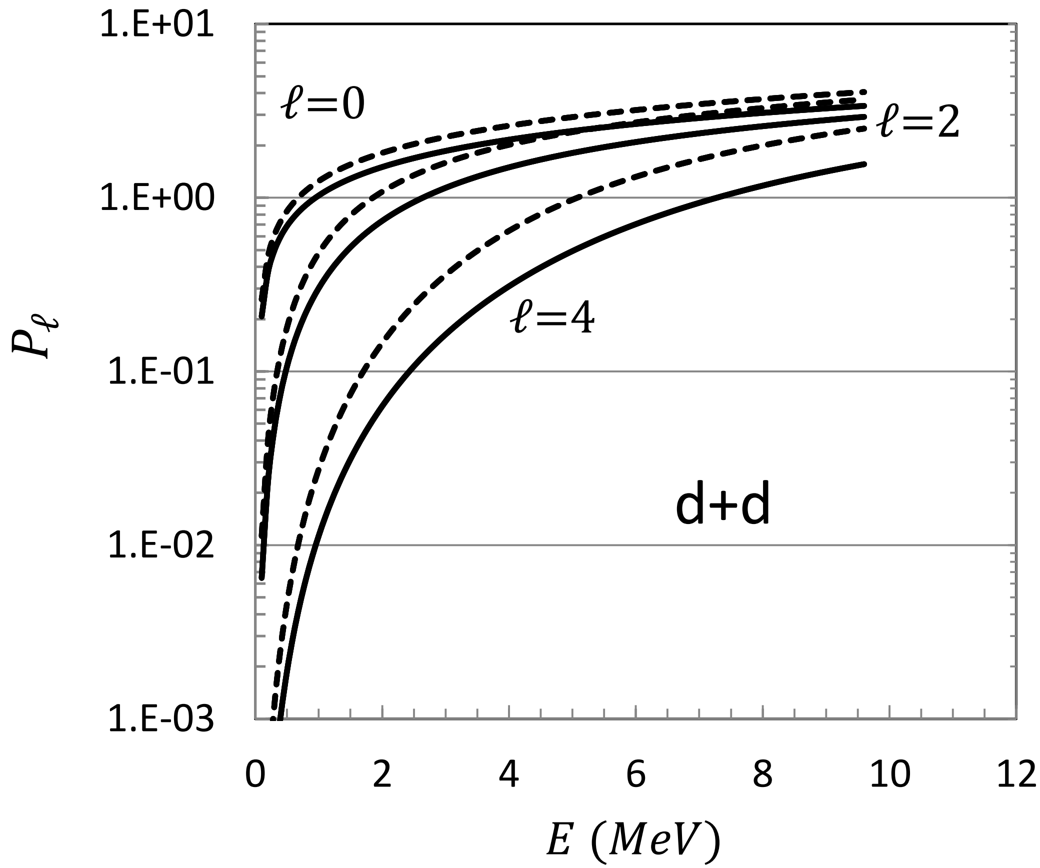


Astrophysical S factor: $S(E)=\sigma(E)*E*\exp(2\pi\eta)$ (Units: $E*L^2$: MeV-barn)

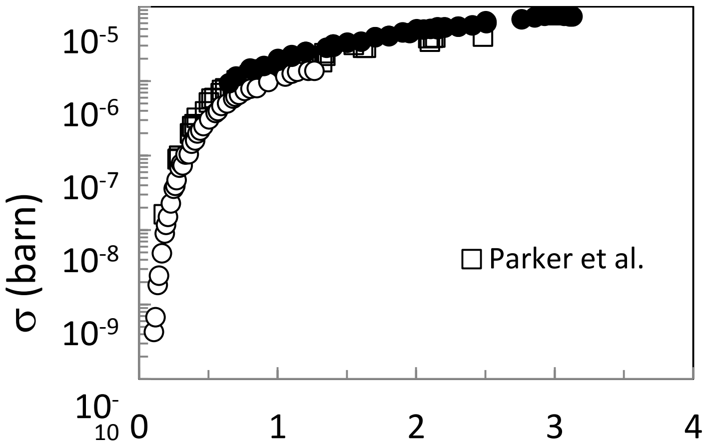
- removes the coulomb dependence → only nuclear effects
- weakly depends on energy → $\sigma(E)\approx S_0\exp(-2\pi\eta)/E$

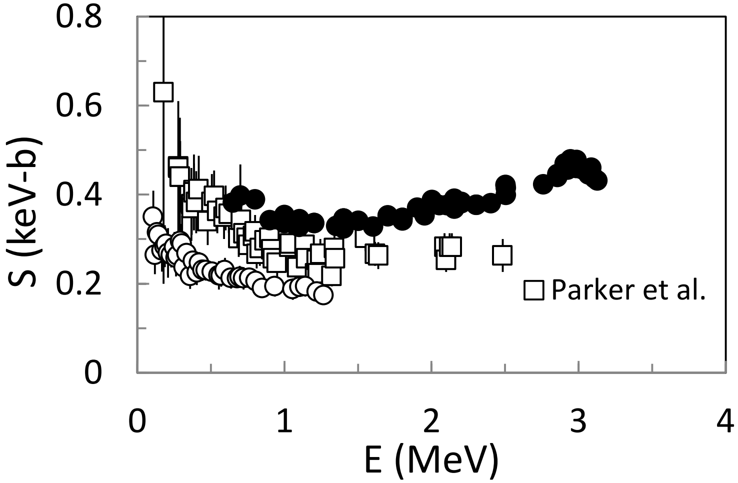
Penetration factor P_ℓ for d+d (2 different radii, a=5 fm and a=6 fm)

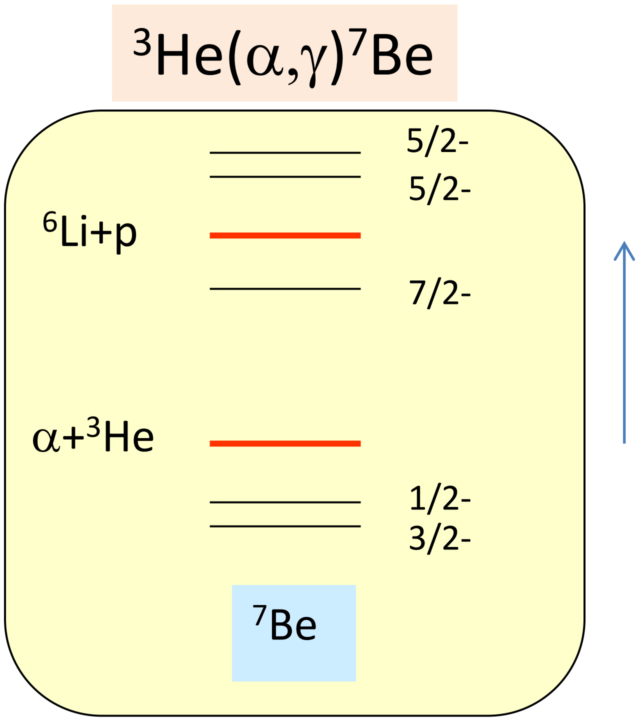
- Typical Coulomb effect
- depends on the radius
- strongly depends on energy E, and on angular momentum ℓ
- $P_\ell(E) \sim \exp(-2\pi\eta)$ for $\ell = 0$



Non resonant: $S(E)=\sigma(E)*E*\exp(2\pi\eta)$ approximately constant





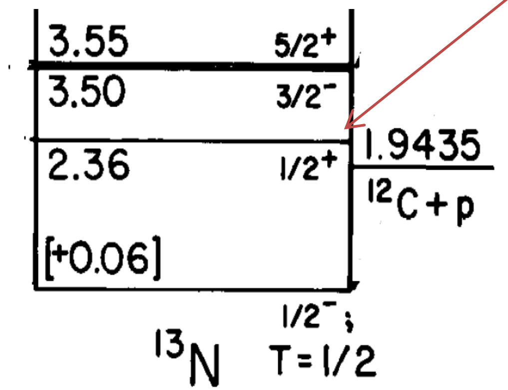


No resonance with low J

Resonances: the Breit-Wigner approximation

General properties of a resonance:

- spin J_R
- Energy E_R
- Entrance width (here: particle width)
- Output width (here: gamma width)



$J_R = 1/2^+$
$E_R = 0.42$ MeV
$\Gamma_p = 32$ keV
$\Gamma_\gamma = 0.5$ eV

Breit-Wigner approximation near a resonance: $E \approx E_R$:

$$\sigma(E) \approx \frac{\pi}{k^2}(2J_R + 1)\frac{\Gamma_\gamma(E)\Gamma_p(E)}{(E - E_R)^2 + \Gamma(E)^2/4}$$

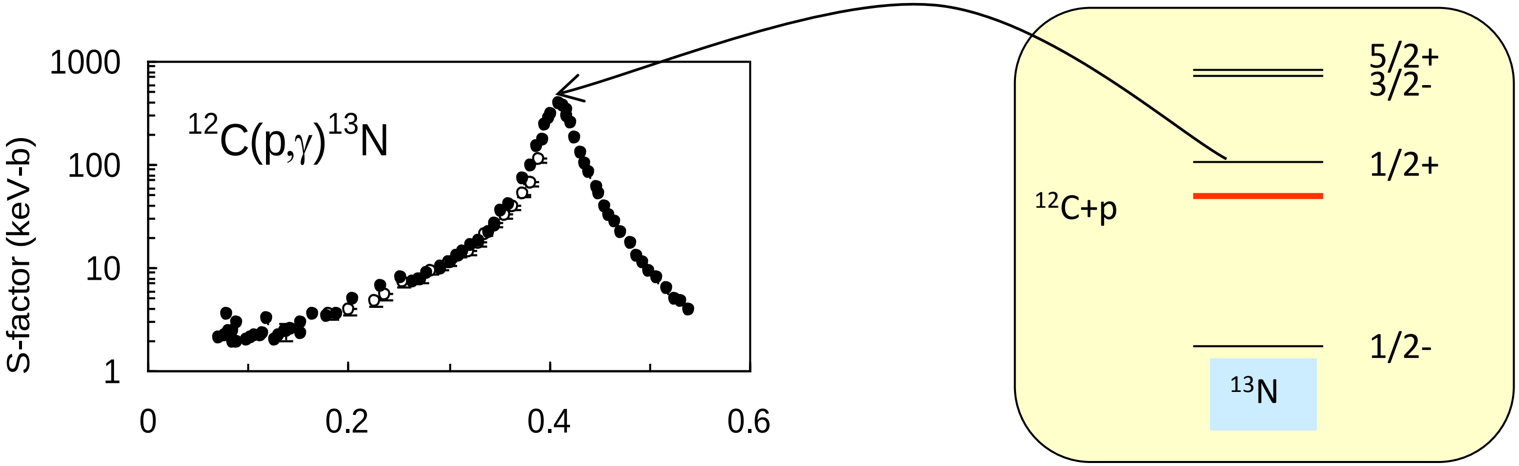
Valid for the resonant partial wave J_R → possible contribution of other partial waves

Two typical examples:

- ^{12}C(p,γ)^{13}N: a resonance is present in the dominant partial wave $(\ell = 0)$
- ^{7}Be(p,γ)^{8}B: resonance in the $\ell = 1$ partial wave → superposition of resonant and non-resonant contributions

Example 1: $^{12}C(p,\gamma)^{13}N$

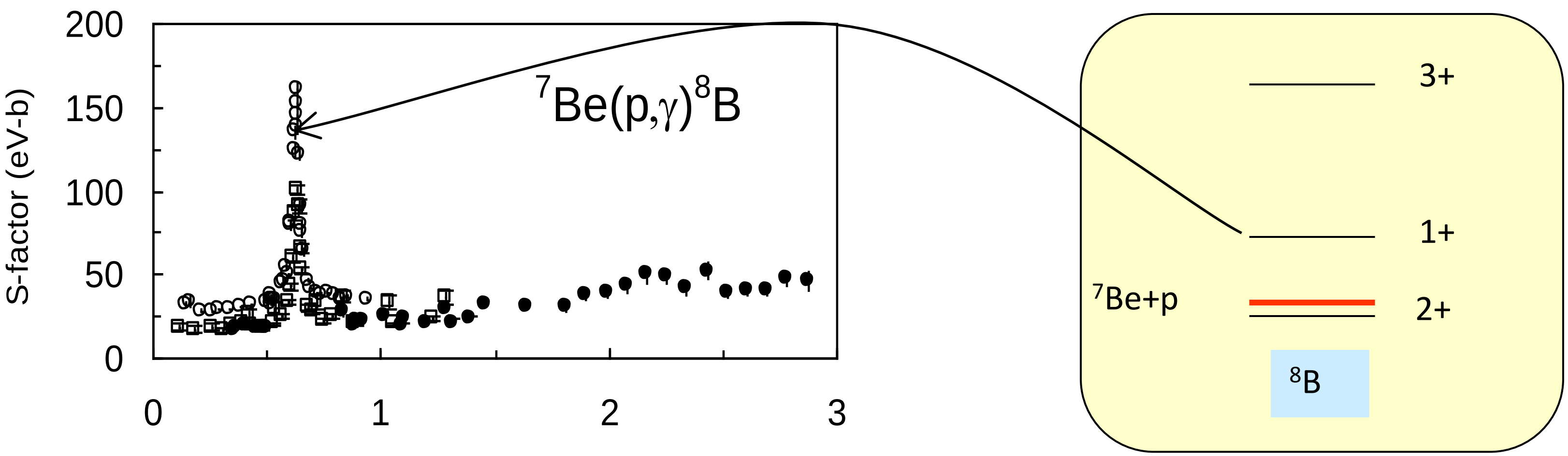
- $J_f=1/2^-$
- E1 → $J_i=1/2^+$, $\ell_i=0$ → dominant
  $J_i=3/2^+$, $\ell_i=2$
- Resonance for $J_i=1/2^+$ → enhancement
- → Two effects make $J_i=1/2^+$ dominant



Example 2: $^7Be(p,\gamma)^8B$

- $J_f=2^+$
- E1 → $J_i=1^-, \ell_i=0,2$ → $1^-, 2^-$ dominant
  $J_i=2^-, \ell_i=0,2,4$
  $J_i=3^-, \ell_i=2,4$
- Resonance for $J_i=1^+$
  - only E2 or M1 are possible
  - $\ell_i=1,3$

→ limited effect of the 1^+ resonance
Both contributions must be treated separately



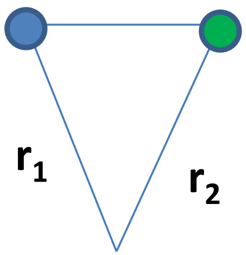
5. Radiative capture in the potential model

5. Radiative capture in the potential model

Potential model: two structureless particles (=optical model, without imaginary part)

- Calculations are simple
- Physics of the problem is identical in other methods
- Spins are neglected

- $\boldsymbol{R_{cm}}$=center of mass, $\boldsymbol{r}$ =relative coordinate



$$\boldsymbol{r_1} = \boldsymbol{R_{cm}} - \frac{A_2}{A}\boldsymbol{r}$$
$$\boldsymbol{r_2} = \boldsymbol{R_{cm}} + \frac{A_1}{A}\boldsymbol{r}$$

- Initial wave function: $\Psi^{\ell_i m_i}(\boldsymbol{r}) = \frac{1}{r} u_{\ell_i}(r) Y_{\ell_i}^{m_i}(\Omega)$, energy E^{ℓ_i}=scattering energy E

  Final wave function: $\Psi^{\ell_f m_f}(\boldsymbol{r}) = \frac{1}{r} u_{\ell_f}(r) Y_{\ell_f}^{m_f}(\Omega)$, energy E^{ℓ_f}

  The radial wave functions are given by:

$$-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right)u_\ell + V(r)u_\ell = E^\ell u_\ell$$

5. Radiative capture in the potential model

- Schrödinger equation: $-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right)u_\ell + V(r)u_\ell = E^\ell u_\ell$
- Typical potentials:
  - coulomb =point-sphere
  - nuclear: Woods-Saxon, Gaussian
    with parameters adjusted on important properties (bound-state energy, phase shifts, etc.)

- Potentials can be different in the initial and final states

- Numerical method: Numerov (finite differences, based on a step h and N points)
  - $u_\ell(0) = 0$
  - $u_\ell(h) = 1$
  - $u_\ell(2h)$ is determined from $u_\ell(0)$ and $u_\ell(h)$
  - … → $u_\ell(Nh)$

- $u_\ell(r)$ is adjusted on $u_\ell(r) = F_J(kr)\cos\delta_J + G_J(kr)\sin\delta_J$ at large distances
  → phase shift
  → the wave function is available in a numericall format
  → any matrix element can be computed

5. Radiative capture in the potential model

- Electric operator for two particles:

$$\mathcal{M}_\mu^{E\lambda} = e\left(Z_1|\boldsymbol{r_1} - \boldsymbol{R_{cm}}|^\lambda\, Y_\lambda^\mu\left(\Omega_{r_1-R_{cm}}\right) + Z_2|\boldsymbol{r_2} - \boldsymbol{R_{cm}}|^\lambda\, Y_\lambda^\mu\left(\Omega_{r_2-R_{cm}}\right)\right)$$

which provides

$$\mathcal{M}_\mu^{E\lambda} = e\left[Z_1\left(-\frac{A_2}{A}\right)^\lambda + Z_2\left(\frac{A_1}{A}\right)^\lambda\right] r^\lambda Y_\lambda^\mu(\Omega_r) = eZ_{eff}\, r^\lambda Y_\lambda^\mu(\Omega_r)$$

- Matrix elements needed for electromagnetic transitions

$$< \Psi^{J_f m_f}\left|\mathcal{M}_\mu^{E\lambda}\right|\Psi^{J_i m_i} > = eZ_{eff} < Y_{J_f}^{m_f}\left|Y_\lambda^\mu\right|Y_{J_i}^{m_i} > \int_0^\infty u_{J_i}(r)\, u_{J_f}(r) r^\lambda dr$$

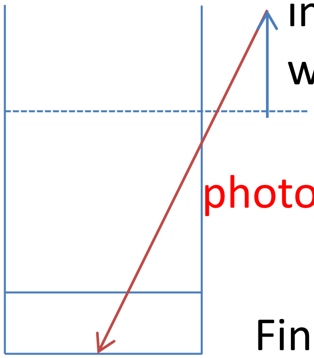
- Reduced matrix elements:

$$< \Psi^{J_f}\left\|\mathcal{M}^{E\lambda}\right\|\Psi^{J_i} > = eZ_{eff} < J_f 0 \lambda 0|\, J_i 0 >$$
$$\times \left(\frac{(2J_i+1)(2\lambda+1)}{4\pi(2J_f+1)}\right)^{1/2} \int_0^\infty u_{J_i}(r)\, u_{J_f}(r) r^\lambda dr$$

→ simple one-dimensional integrals

Assumptions:

- spins zero: $\ell_i = J_i, \ell_f = J_f$
- given values of J_i, J_f, λ



Integrated cross section

$$\sigma_\lambda(E) = \frac{8\pi}{k^2}\frac{e^2}{\hbar c} Z_{eff}^2 k_\gamma^{2\lambda+1} F(\lambda, J_i, J_f) \left| \int_0^\infty u_{J_i}(r,E)\, u_{J_f}(r) r^\lambda dr \right|^2$$

with

- $Z_{eff} = Z_1 \left(-\frac{A_2}{A}\right)^\lambda + Z_2 \left(\frac{A_1}{A}\right)^\lambda$
- $F(\lambda, J_i, J_f) =< J_i \lambda 0\, 0 | J_f 0 > (2J_i + 1) \frac{(\lambda+1)(2\lambda+1)}{\lambda (2\lambda+1)!!^2}$
- $k_\gamma = \frac{E - E_f}{\hbar c}$

Normalization

- final state (bound): normalized to unity $u_J(r) \rightarrow C\exp(-k_B r)$
- initial state (continuum): $u_J(r) \rightarrow F_J(kr)\cos\delta_J + G_J(kr)\sin\delta_J$

Test: $\lambda = 0$ provides $\sigma_\lambda(E) = 0$ (orthogonality of the wave functions)

Integrated vs differential cross sections

- Total (integrated) cross section:

$$\sigma(E) = \sum_{\lambda} \sigma_{\lambda}(E)$$

→ no interference between the multipolarities

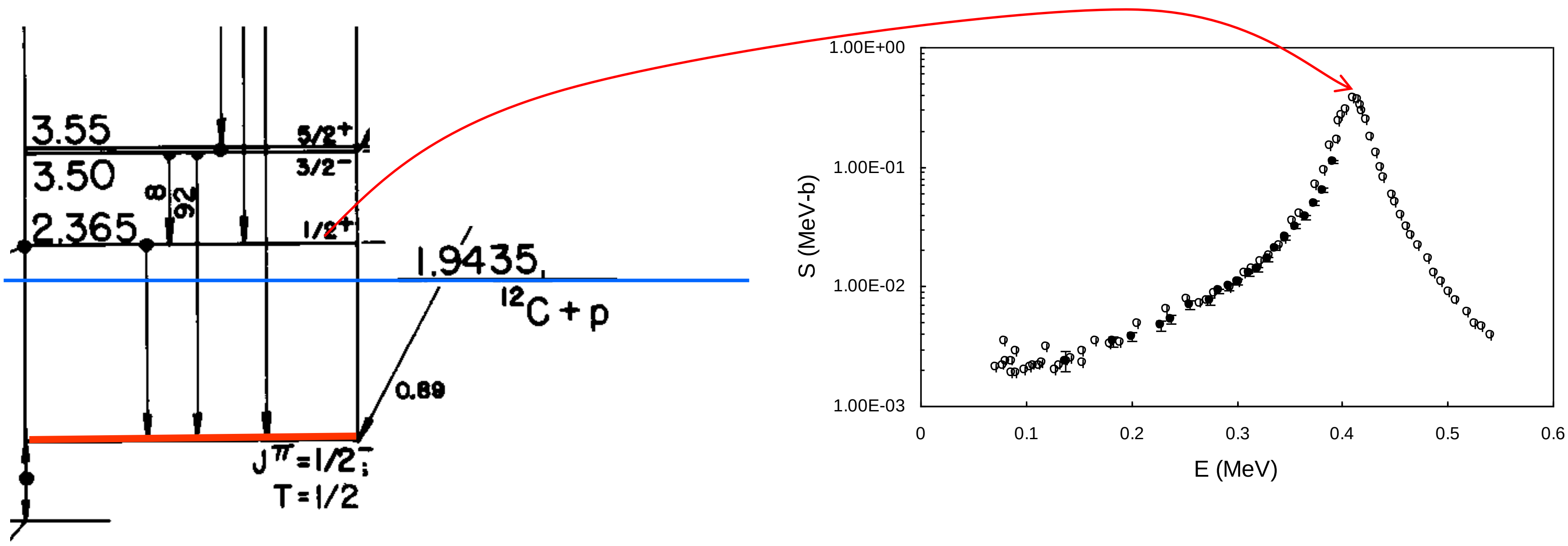
- Differential cross section:

$$\frac{d\sigma}{d\theta} = \left| \sum_{\lambda} a_{\lambda}(E) P_{\lambda}(\theta) \right|^2$$

$P_{\lambda}(\theta)$=Legendre polynomial
→ interference effects
→ angular distributions are necessary to separate the multipolarities (in general one multipolarity is dominant)

Example: $^{12}C(p,\gamma)^{13}N$

- First reaction of the CNO cycle
- Well known experimentally
- Presents a low energy resonance (l=0 → J=1/2+)

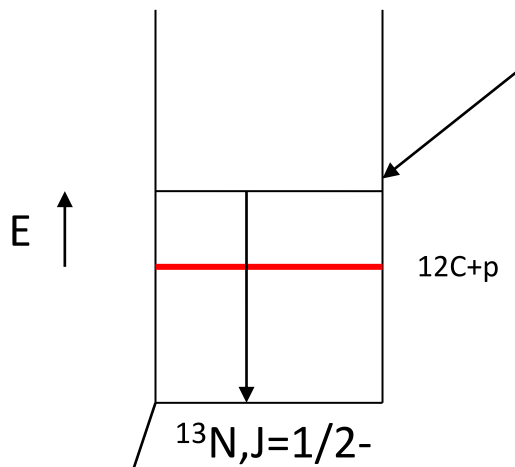


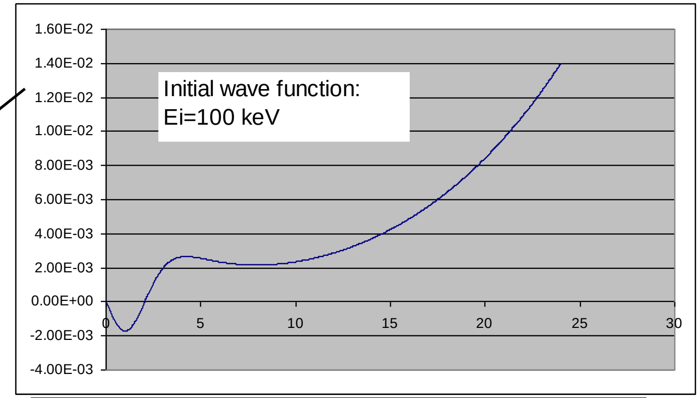
Potential : $V= -55.3*\exp(-(r/2.70)^2)$ (final state)

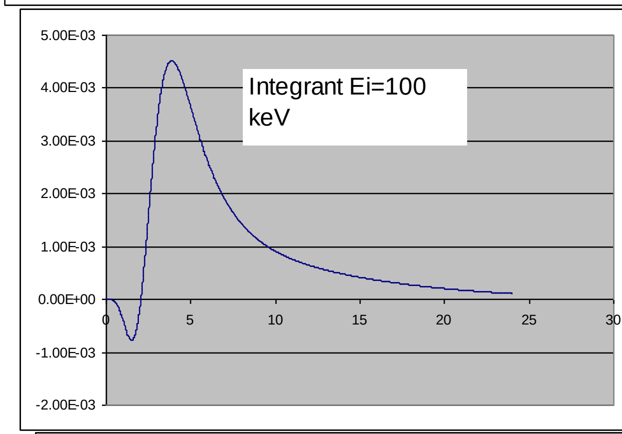
$-70.5*\exp(-(r/2.70)^2)$ (initial state)

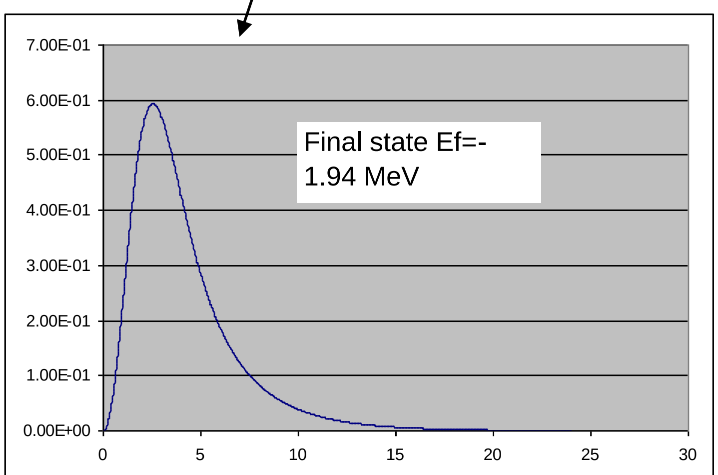
5. Radiative capture in the potential model

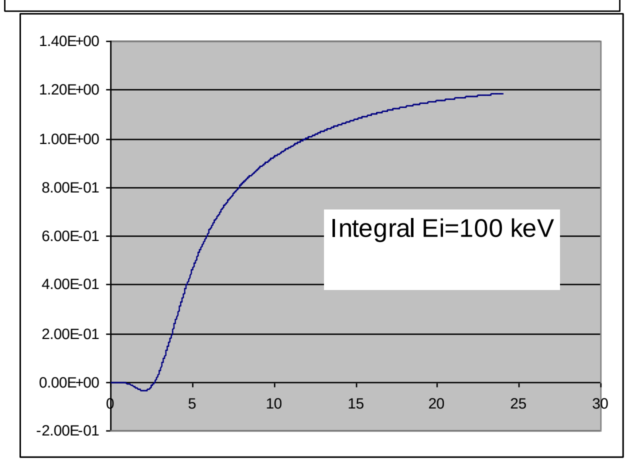
Final state: $J_f=1/2^-$
Initial state: $l_i=0 \rightarrow J_i=1/2^+$
$\rightarrow$ E1 transition $1/2^+ \rightarrow 1/2^-$



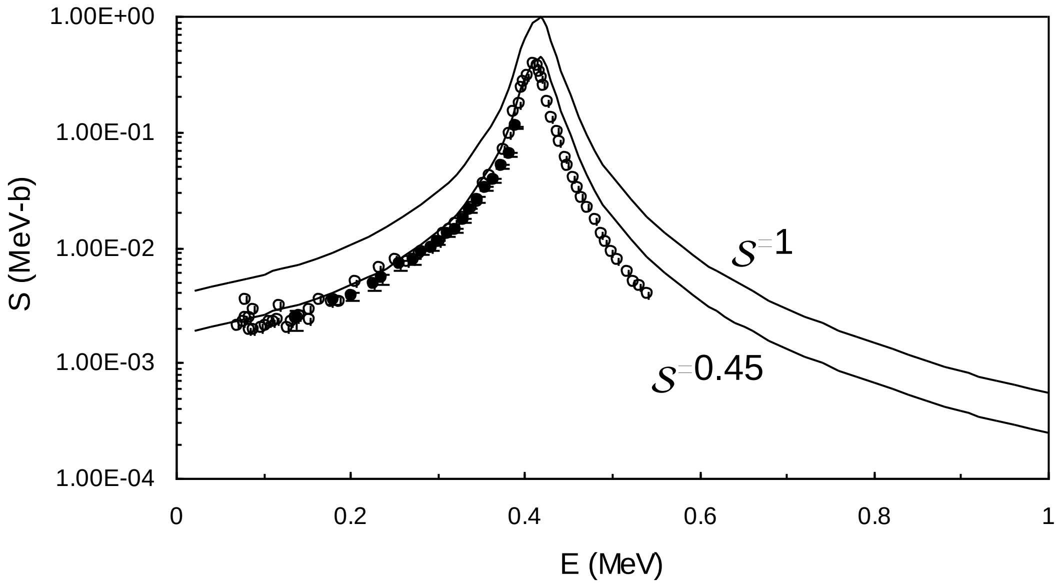








The calculation is repeated at all energies



Necessity of a spectroscopic factor *S*

Assumption of the potential model: $^{13}N = ^{12}C + p$
In reality $^{13}N = ^{12}C + p \oplus ^{12}C^{*} + p \oplus ^{9}Be + \alpha \oplus \ldots$

➔ to simulate the missing channels: $u_f(r)$ is replaced by $S^{1/2} u_f(r$
S=spectroscopic factor

Other applications: $^{7}Be(p,\gamma)^{8}B$, $^{3}He(\alpha,\gamma)^{7}Be$, etc…

Internal-external contributions

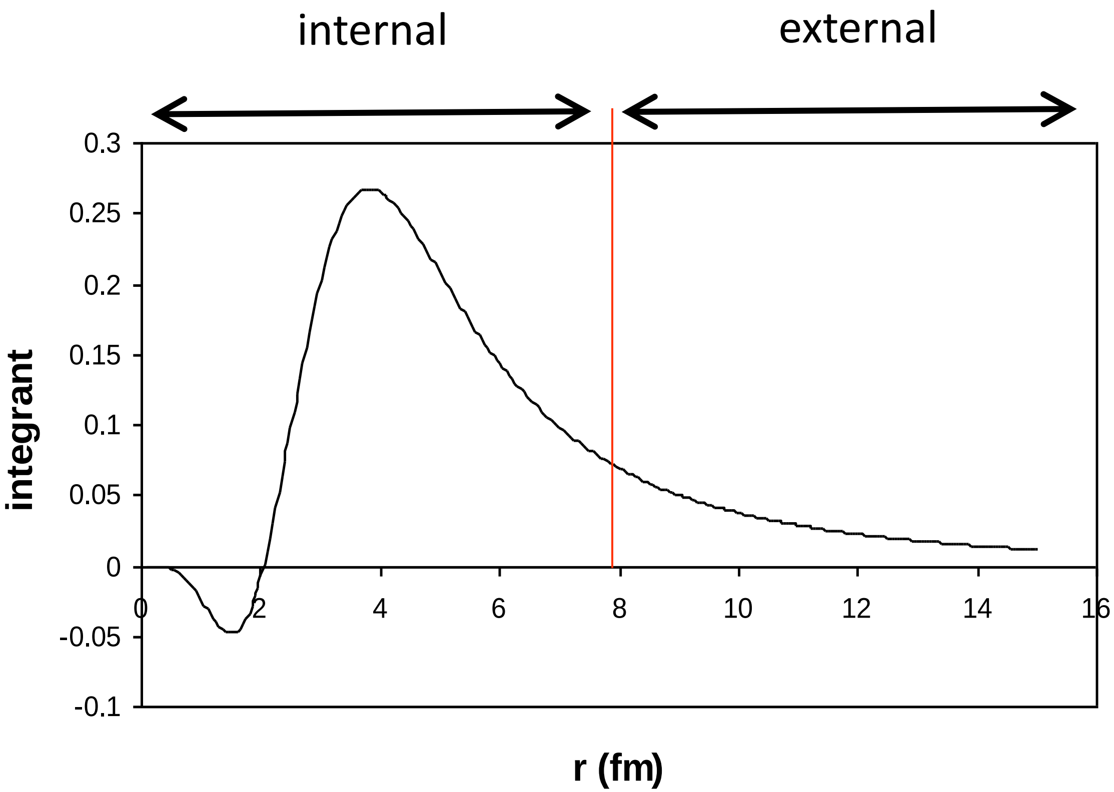
$$\sigma_\lambda(E) = \frac{8\pi}{k^2}\frac{e^2}{\hbar c} Z_{eff}^2 k_\gamma^{2\lambda+1} F(\lambda, J_i, J_f) \left| \int_0^\infty u_{J_i}(r,E)\, u_{J_f}(r) r^\lambda dr \right|^2$$

- initial scattering wave function $u_J(r) \to F_J(kr)\cos\delta_J + G_J(kr)\sin\delta_J$
- final bound-state wave function: binding energy E_B
  $u_f(r) \to \boldsymbol{C} W_{-\eta_B,\ell+1/2}(2k_B r)$, $W_{-\eta_B,\ell+1/2}(x)$ is the Whittaker function

  $W_{-\eta_B,\ell+1/2}(2k_B r) \to \exp(-k_B r)$

  $\boldsymbol{C}$=Aymptotic Normalization Constant (ANC)
  with $k_B = \sqrt{2\mu E_B}/\hbar^2$

Internal and external components of the integrant



Internal

$$M_{int} = \int_0^a u_{J_i}(r,E)\, u_{J_f}(r) r^\lambda dr$$

external

$$M_{ext} = \int_a^\infty u_{J_i}(r,E)\, u_{J_f}(r) r^\lambda dr$$

asymptotic forms are used

a= typical radius (~range of the nuclear interaction)

Capture cross section: $\sigma \sim |M_{int} + M_{ext}|^2$

5. Radiative capture in the potential model

Capture cross section: $\sigma \sim |M_{int} + M_{ext}|^2$

Internal part: $M_{int} = \int_0^a u_{J_i}(r,E)\, u_{J_f}(r) r^\lambda dr$

External part : $M_{ext} = \int_a^\infty u_{J_i}(r,E)\, u_{J_f}(r) r^\lambda dr$

with $u_{J_f}(r) = C W_{-\eta_B,\ell+1/2}(2k_B r)$

$u_{J_i}(r,E) = F_J(kr)\cos\delta_J + G_J(kr)\sin\delta_J$

At low energies: $\delta_J \approx 0$ → the ANC C determines the external contribution

Different options

- $\boldsymbol{M_{int}}$ **dominant,** $\boldsymbol{M_{ext}}$ **negligible**: high energies, resonant reactions
- $\boldsymbol{M_{int}}$ **and** $\boldsymbol{M_{ext}}$ **important**
- $\boldsymbol{M_{int}}$ **negligible,** $\boldsymbol{M_{ext}}$ **dominant** : low binding energies

  → « peripheral reaction »

  → amplitude of cross section determined by the ANC

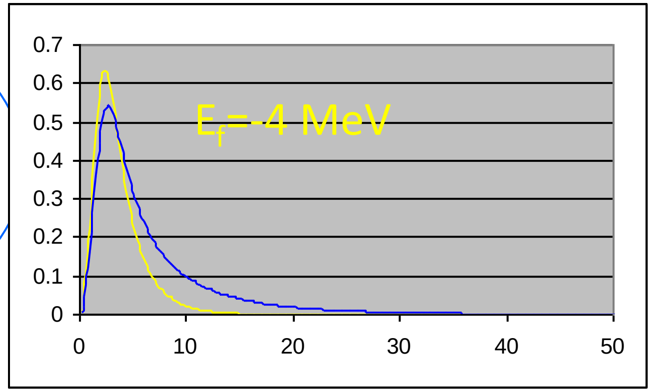
  → energy dependence determined by the Coulomb functions

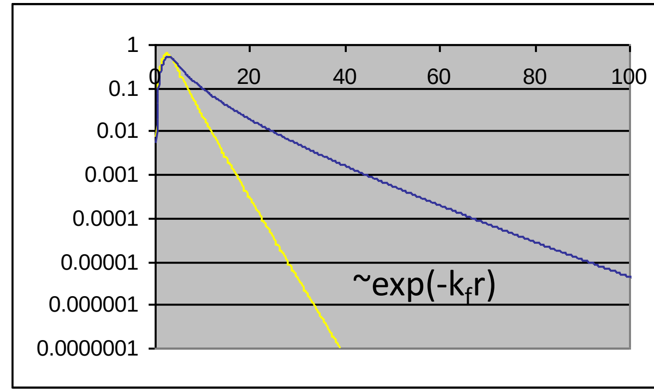
  → measurement of the ANC provides the cross section at low energies

5. Radiative capture in the potential model

Bound state:
E_f=-0.137 MeV

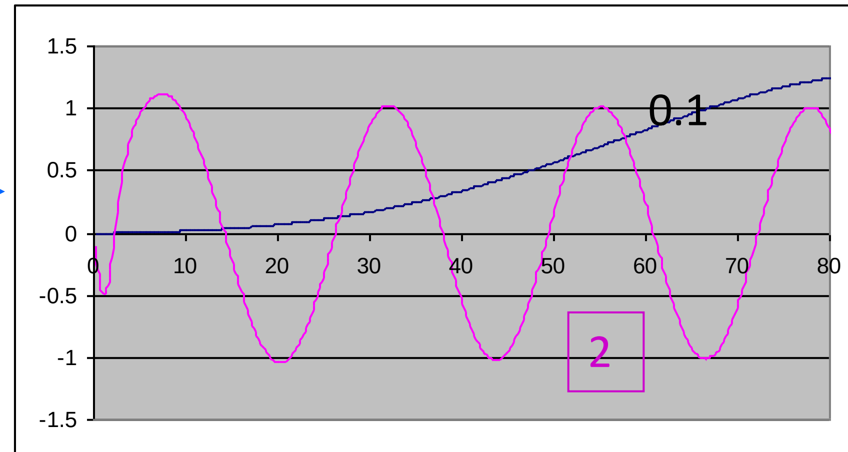
wave function $u_{J_f}(r)$





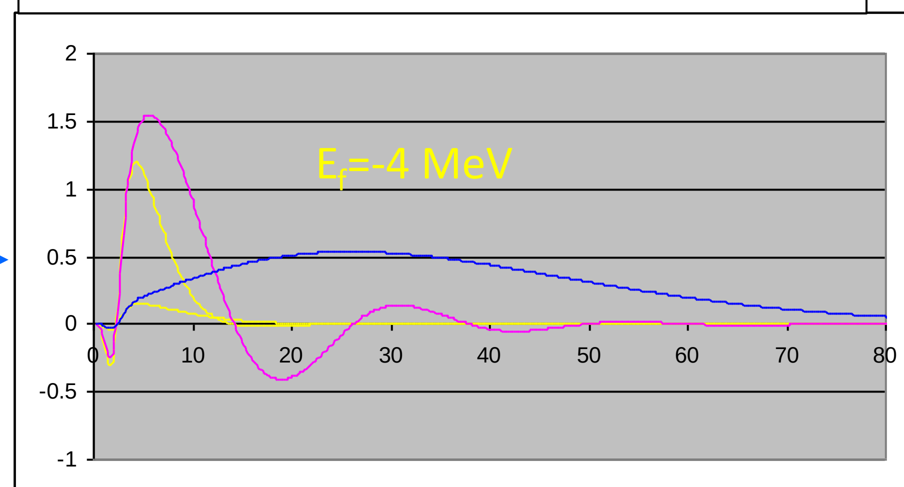
Initial state E_i=0.1 and 2 MeV

wave function $u_{J_i}(r,E)$



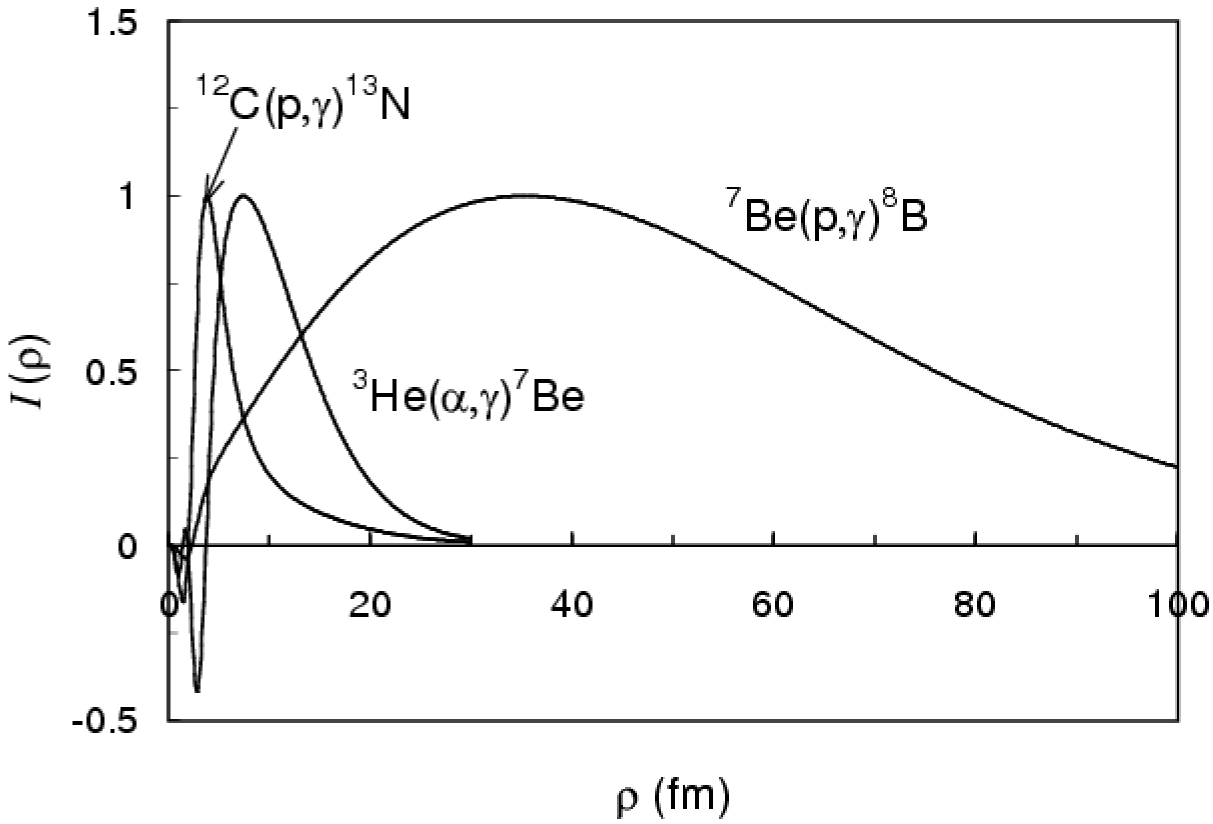
$^7Be(p,\gamma)^8B$

Integrant
$u_{J_i}(r,E)u_{J_f}(r)r^3$



Integrant for 3 reactions at low energies

- $^{7}Be(p,\gamma)^{8}B$ external (low binding energy)
- $^{12}C(p,\gamma)^{13}N$ internal: resonant reaction
- $^{3}He(\alpha,\gamma)^{7}Be$: both components are important



6. Conclusion

- Radiative capture important in astrophysics

- Low cross sections
  - Electromagnetic process
  - Astrophysics: low energies → Coulomb barrier reduces the cross section

- Potential model
  - Simple calculations
  - Illustration of the internal-external components