Microscopic cluster models II

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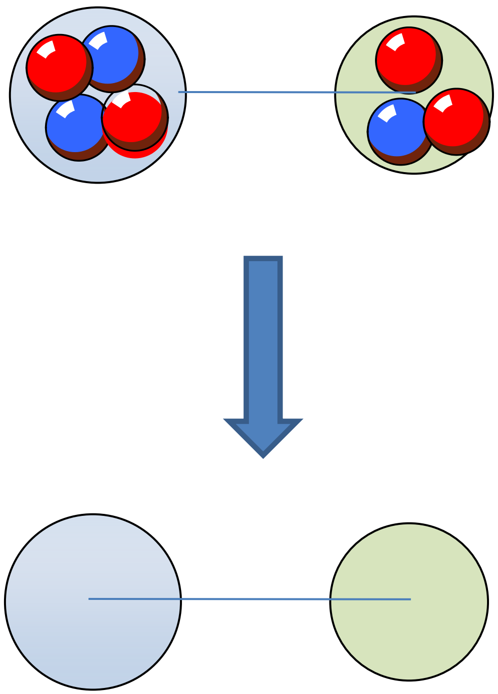
1. Introduction
2. Important concepts: isospin, antisymmetrization
3. The nucleon-nucleon interaction
4. The shell model
5. Overview of microscopic models
6. The Generator Coordinate Method (GCM)
7. Reactions with the GCM
8. Forbidden states
9. Some examples

8. Forbidden states

8. Forbidden states

1. From microscopic to non-microscopic models



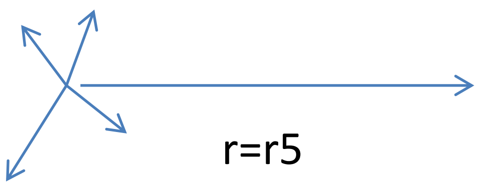
microscopic: $H = \sum_i T_i + \sum_{j>i} V_{ij}$

non-microscopic: $H = T_r + V(r)$

Question: how to account for microscopic effects in a nucleus-nucleus-potential $V(r)$?
➔ forbidden states

2. Effect of the antisymmetrization operator

Example: α+nucleon



α particle: $\phi_\alpha = \det|\varphi\, n\uparrow\ \ \varphi n\downarrow\ \ \varphi p\uparrow \varphi n\downarrow|$
$\varphi(r) = \exp(-r^2/2b^2)$

α+n system
without $\mathcal{A}$: $\Psi = \det|\varphi\, n\uparrow\ \ \varphi n\downarrow\ \ \varphi p\uparrow \varphi n\downarrow|\ \ g(r)n\uparrow = \phi_\alpha\phi_n g(r)$

with $\mathcal{A}$: $\Psi = \det|\varphi\, n\uparrow\ \ \varphi n\downarrow\ \ \varphi p\uparrow \varphi n\downarrow\ \ g(r)n\uparrow| = \mathcal{A}\phi_\alpha\phi_n g(r)$

if the relative function is equal to a 0s orbital: $g(r) = \varphi(r) = \exp(-r^2/2b^2)$
- $\phi_\alpha\phi_n\varphi(r) \neq 0$
- $\mathcal{A}\phi_\alpha\phi_n\varphi(r) = 0$

→ **specific effect of antisymmetrization**
→ $\varphi(r) = \exp(-r^2/2b^2)$ is a « forbidden » state for the α+n system ($\ell = 0$)

3. General definition

Function $\chi_{\ell n}(r)$ such that

$$\mathcal{A}\phi_1\phi_2\chi_{\ell n}(r) = 0$$

with $\chi_{\ell n}(r) \neq 0$

- Assumes shell-model wave functions (with $b_1 = b_2 = b$)
  If $b_1 \neq b_2$: « almost forbidden states »
- Depends on the system
- Depends on the angular momentum ℓ

Examples

- α+n $\ell = 0$: 1 fs
  $\ell > 0$: 0 fs

- α+α $\ell = 0$: 2 fs
  $\ell = 2$: 1 fs
  $\ell > 2$: 0 fs

4. Consequences on the nucleus-nucleus interaction

- Forbidden states are defined by $\Psi^{\ell}_{fs} = \mathcal{A}\phi_1\phi_2\chi_{\ell n}(r) = 0$

- In microscopic models
  - $\Psi^{\ell} = \mathcal{A}\phi_1\phi_2 g_{\ell}$
  - $< \Psi^{\ell}_{fs} | \Psi^{\ell} > = 0$ for any wave function Ψ^{ℓ} → Ψ^{ℓ} is orthogonal to the forbidden states

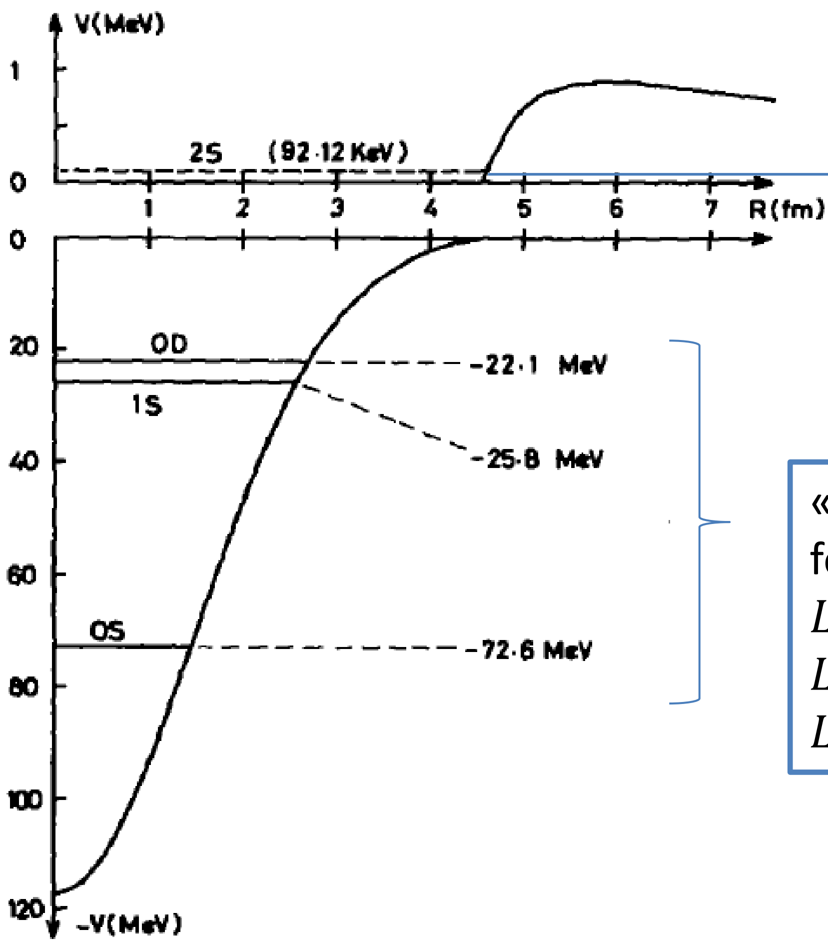
- In non-microscopic models
  - $\Psi^{\ell} = g_{\ell}$
  - forbidden states can be simulated by imposing by imposing $< g_{\ell} | \chi_{\ell n} > = 0$

- Nucleus-nucleus potentials: additional bound states → deep potentials

- Example: Buck potential for $\alpha+\alpha$ $V_N(r)=-122.3*\exp(-(r/2.13)^2)$

8. Forbidden states

Buck potential for α+α : Nucl. Phys A. 275 (1977) 246
Perfectly reproduces the α+α experimental phase shifts up to 20 MeV



« physical » state: gs of ^{8}Be

« unphysical » states: simulate the forbidden states
$L = 0$: 2 f.s. $0S, 1S$
$L = 2$: 1 f.s. $0D$
$L \geq 4$: 0 fs

8. Forbidden states

Other systems:

α+^{16}O: $2n_r + \ell = 8$, ℓ even

$2n_r + \ell = 9$, ℓ odd

^{12}C+^{12}C: $2n_r + \ell = 24$

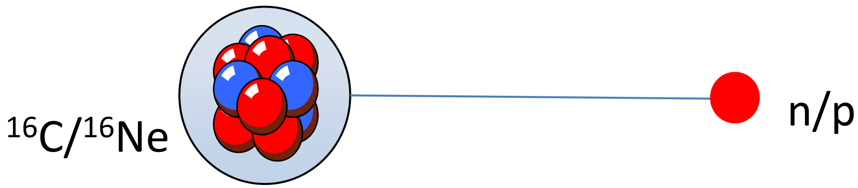
9. Some applications

- Spectroscopy
  - Two-clusters: ^{17}C =^{16}C+n – and ^{17}Na=^{16}Ne+p
  - Three cluster ^{8}He=^{6}He+n+n

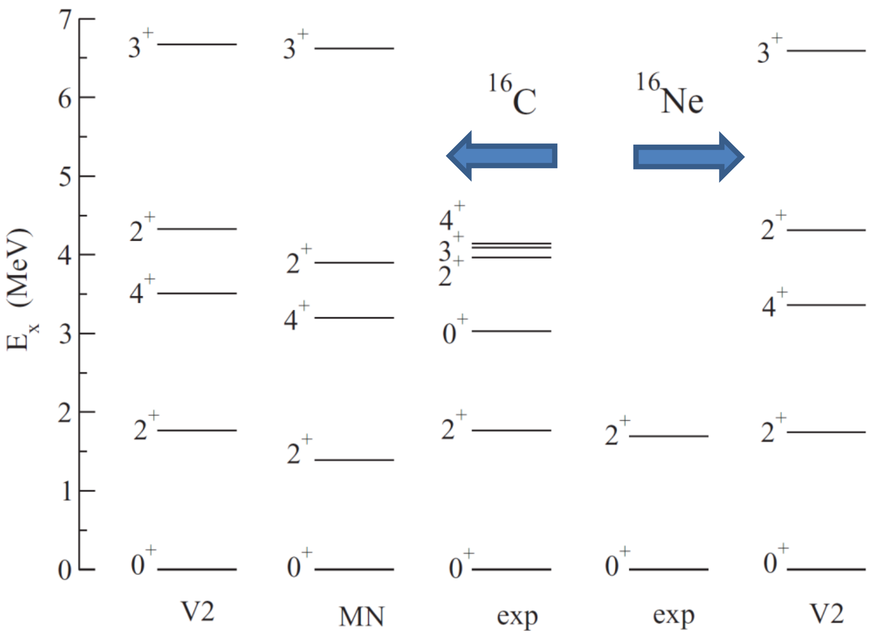
- Reactions
  - Capture: $^{7}Be(p,\gamma)^{8}B$
  - Microscopic CDCC calculations

The ^{17}C and ^{17}Na mirror nuclei

- Ref: N. Timofeyuk, P.D. , Phys. Rev. C81 (2010) 051301
- ^{17}Na unstable (no experimental data but ^{19}Na unstable)
- The mirror ^{17}C nucleus is well known → test with charge symmetry
- Two-cluster systems: ^{16}C+n, ^{16}Ne+p



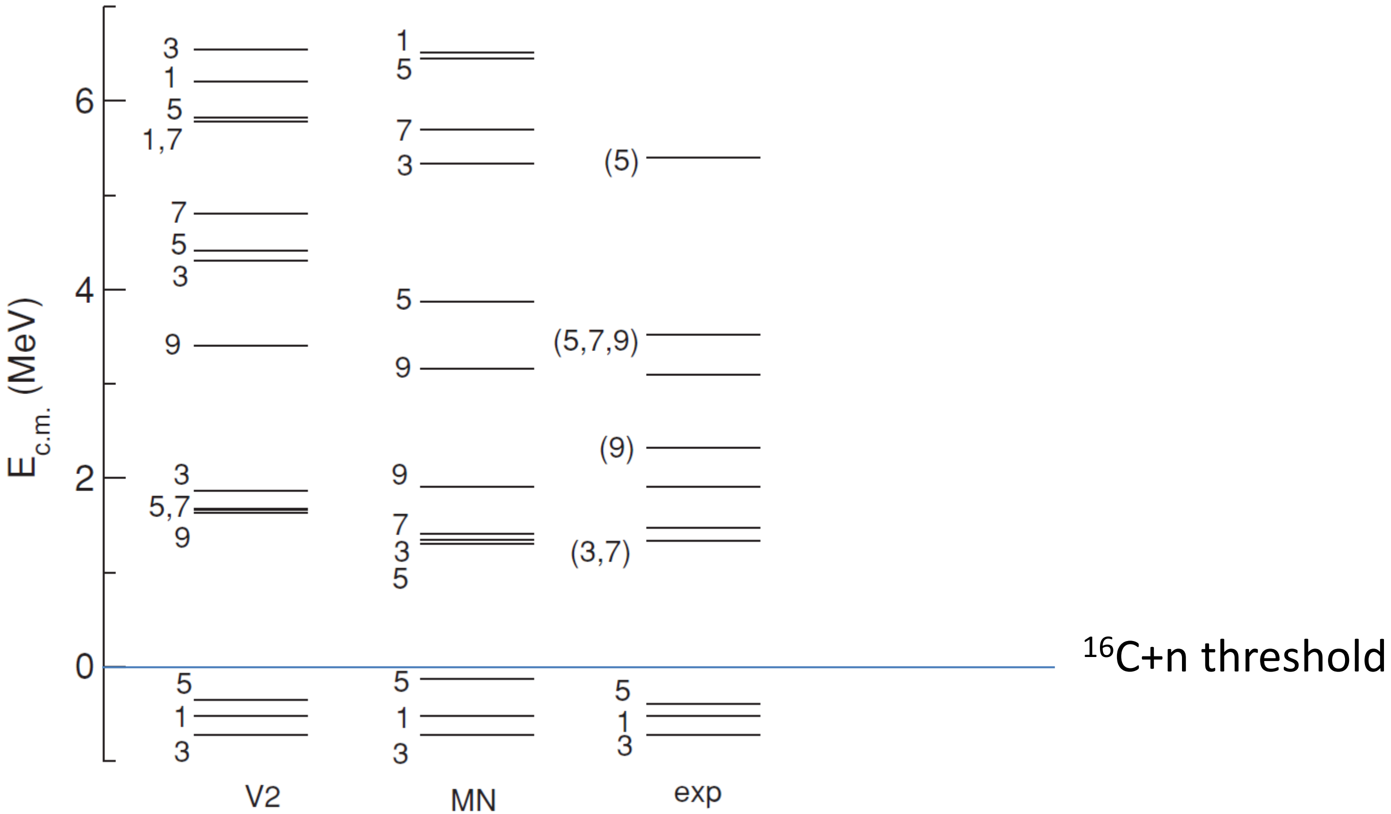
- ^{16}C/^{16}Ne wave functions: 6 protons (s^2p^4), 10 neutrons (s^2,p^6,sd^2) → 15x66=990 SD



Two NN interactions: MN and V2

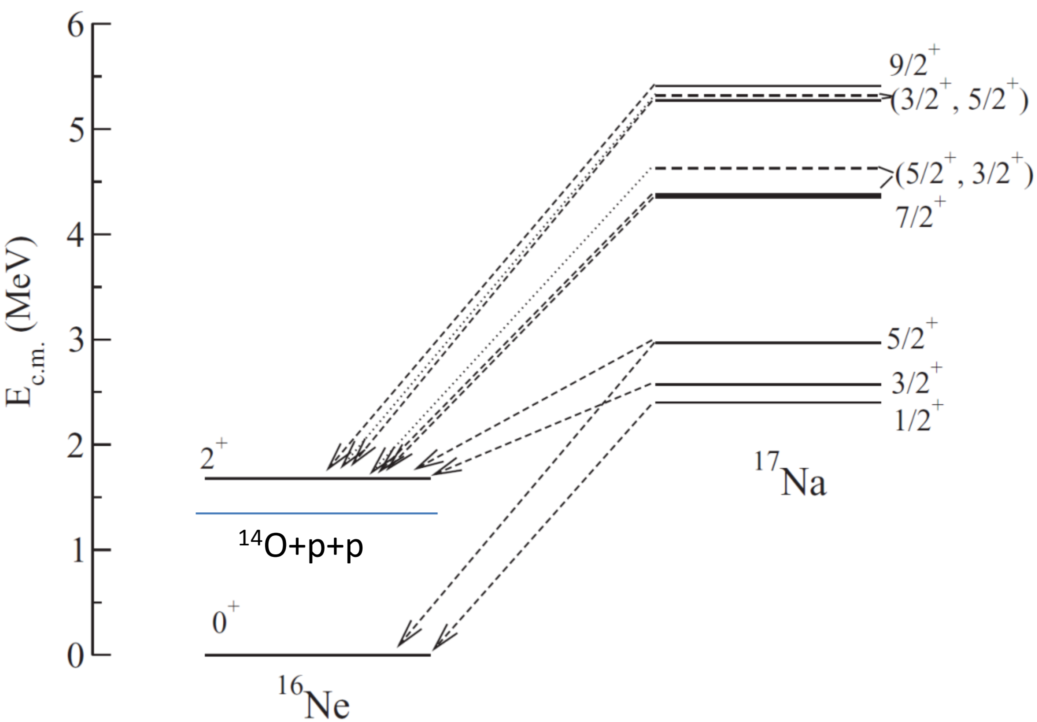
9. Some applications: ^{17}C / ^{17}Na

^{17}C spectrum (positive parity)
Two NN interaction V2 and MN (+spin-orbit)



9. Some applications: ^{17}C / ^{17}Na

^{17}Na spectrum

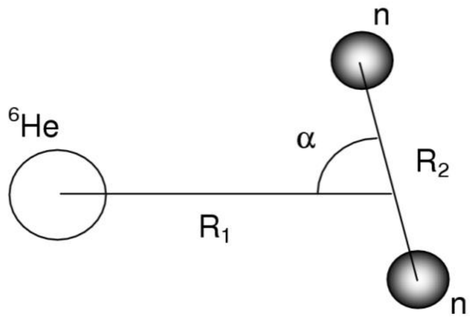


J	E	$\Gamma(0^+)$	$\Gamma(2^+)$
1/2+	2.40	1.36	
3/2+	2.57	0.001	0.024
5/2+	2.97	0.123	0.021
7/2+	4.35	$8\text{x}10^{-8}$	0.025

- all states are unbound → importance of continuum
- ground state: broad ($\ell = 0$) resonance
- excited states: narrow ($\ell = 2$), important decay to the ^{16}Ne(2^+)+p channel
  ➔ 3 proton emittors

The ^8He and ^7He nuclei

- Ref: A. Adahchour, P.D., Phys. Lett. B639 (2006) 447
- Model: $^7He = {}^6He + n$
  $^8He = {}^6He + n + n$

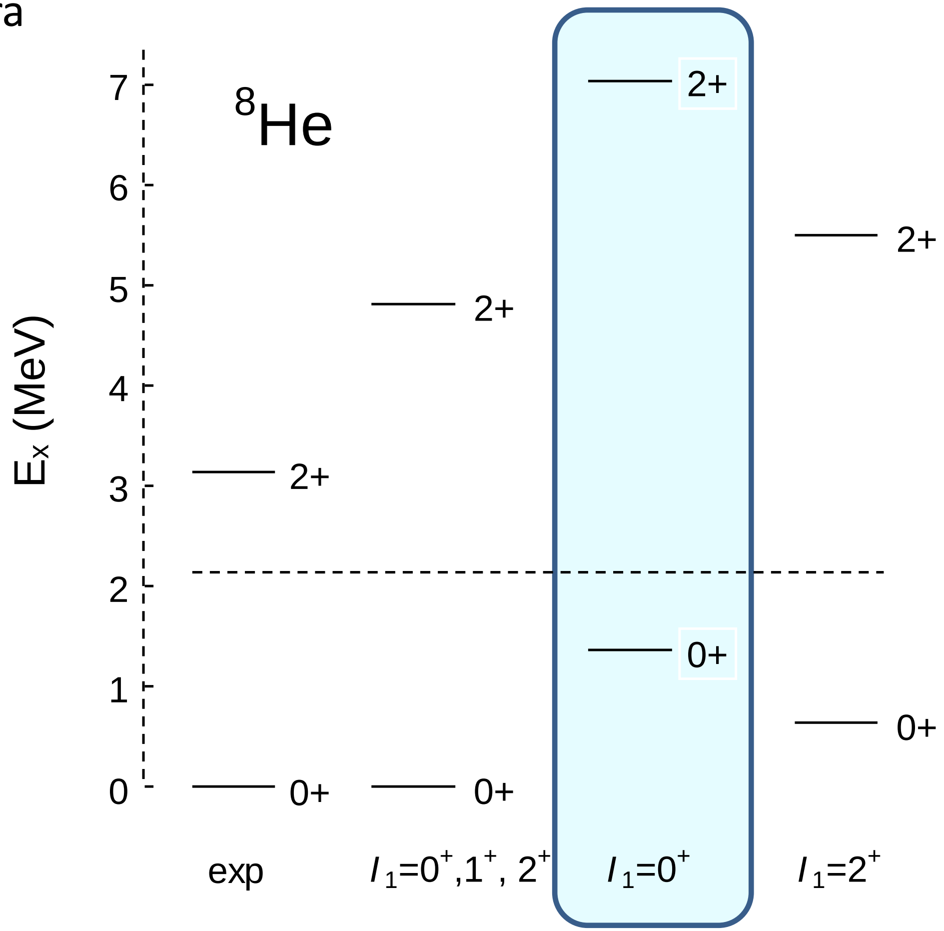


- 3 Generator coordinates for ^8He, 1 for ^7He
- core excitations (p shell) are included

- Basis states $$\Phi_{\gamma K}^{JM\pi}(R_1, R_2, \alpha) = \mathcal{A} P_K^{JM\pi}\, \Phi_{^6He}^{I_1\nu_1}\, \Phi_n^{\frac{1}{2}\nu_2}\, \Phi_n^{\frac{1}{2}\nu_3}$$
- Total wave function $$\Psi_{^8He}^{JM\pi} = \sum_{\gamma K} \sum_{R_1, R_2, \alpha} f_{\gamma K}^{J\pi}(R_1, R_2, \alpha)\, \Phi_{\gamma K}^{JM\pi}(R_1, R_2, \alpha)$$
- Interaction : Minnesota + spin-orbit

9. Some applications: ^{8}He and ^{7}He

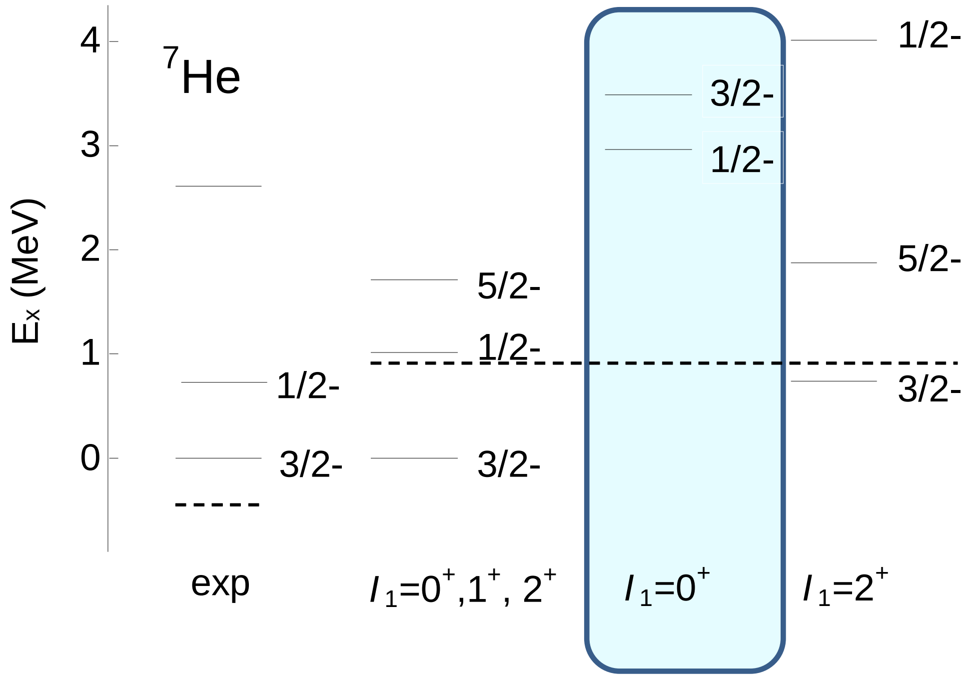
Spectra



- Strong importance of the ^{6}He(2$^+$) excited core (~81% in ^{8}He)
- Korsheninnikov et al., Phys. Rev. Lett. 90 (2003) 082501: ~5/6=0.83

9. Some applications: ^{8}He and ^{7}He

Spectra



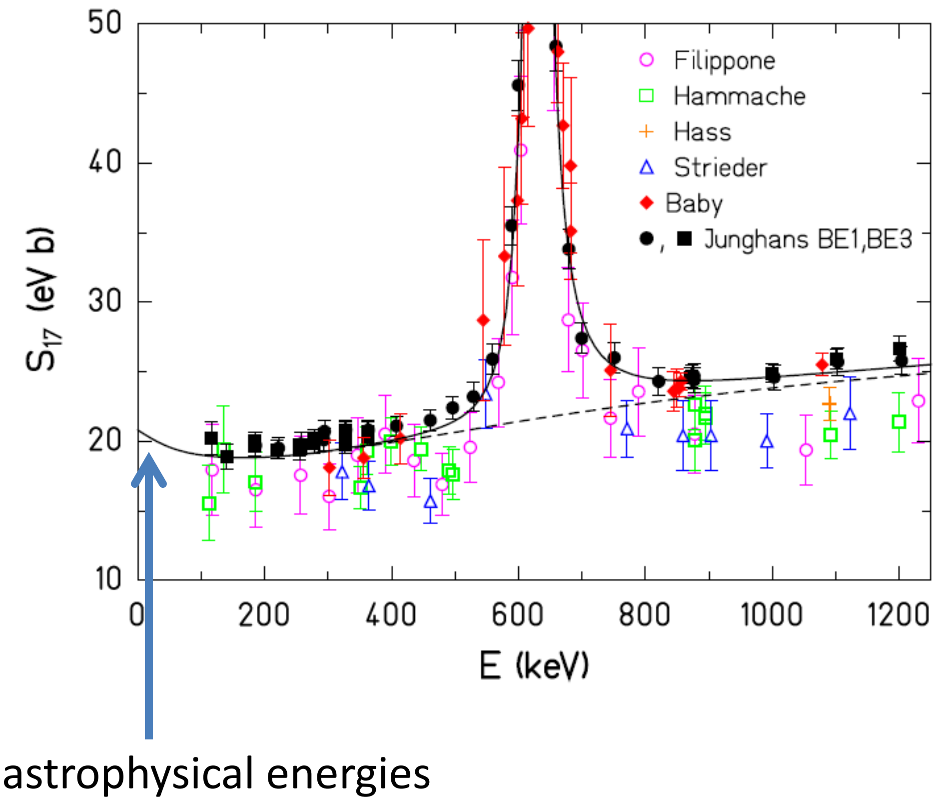
$1/2^-$ and $5/2^-$ excited states are predicted, with ^{6}He(2^+)+n dominant

Previous experiments: K. Markenroth et al. NPA679 (2001) 462
M. Meister et al., PRL 88, 102501 (2002)
$3/2^-$ and $1/2^-$ states, not single-particle states

Recent experiments: A. H. Wuosmaa et al., PRC C **78**, 041302(R) (2008)
$3/2^-$,$1/2^-$,$5/2^-$, but $1/2^-$ at higher energy
Yu. Aksyutina et al, PLBB 679 (2009) 191
SF=0.61 → importance of ^{6}He(2^+)+n

9. Some applications: $^{7}Be(p,\gamma)^{8}B$

- Important for the solar-neutrino problem
- Since 1995, many experiments:
  - Direct (proton beam on a ^{7}Be target)
  - Indirect (Coulomb break-up)
- Extrapolation to zero energy needs a theoretical model (energy dependence)

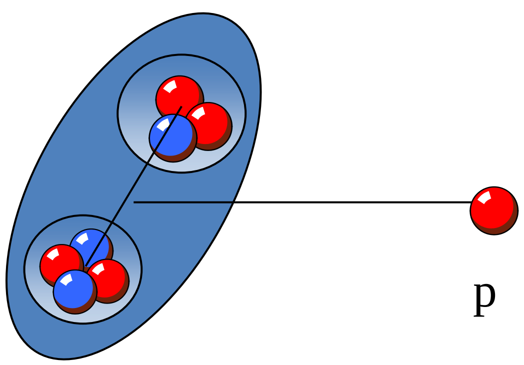


From E. Adelberger et al., Rev. Mod. Phys.83 (2011) 196

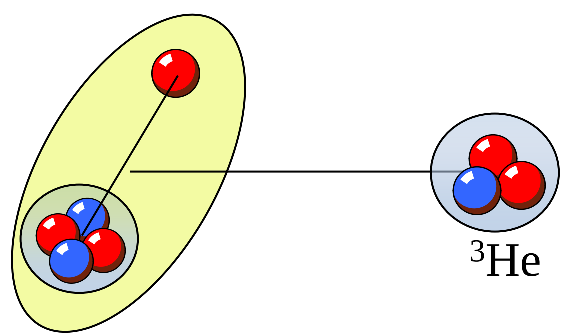
9. Some applications: $^{7}Be(p,\gamma)^{8}B$

- Microscopic cluster calculations: 3-cluster calculations
  - P. D. and D. Baye, Nucl. Phys. A567 (1994) 341
  - P.D., Phys. Rev. C 70, 065802 (2004)

- Includes the deformation of ^{7}Be: cluster structure a+^{3}He
- Includes rearrangement channels $^{5}Li+^{3}He$
- Can be applied to $^{8}B/^{8}Li$ spectroscopy
- Can be applied to $^{7}Be(p,\gamma)^{8}B$ and $^{7}Li(n,\gamma)^{8}Li$



$^{7}Be=\alpha+^{3}He$



$^{5}Li=\alpha+p$

9. Some applications: $^7Be(p,\gamma)^8B$

Spectroscopy of ^8B

	experiment	Volkov	Minnesota
μ (2^+) (μ_N)	1.03	1.48	1.52
$Q(2^+)$ $(e.fm^2)$	6.83 ± 0.21	6.6	6.0
$B(M1, 1^+ \rightarrow 2^+)$ (W.u.)	5.1 ± 2.5	3.4	3.8

Channel components in the ^8B ground state

$^7Be(3/2^-)+p$	47%
$^7Be(1/2^-)+p$	9%
$^5Li(3/2^-)+^3He$	34%
$^5Li(1/2^-)+^3He$	3%

$\Rightarrow$ Important role of the 5+3 configuration

9. Some applications: ^{7}Be(p,γ)^{8}B

- Radiative-capture cross section: $\sigma(J_f, E) \sim \sum_{\lambda\sigma} k_\gamma^{2\lambda+1} \left| < \Psi^{J_f\pi_f} \parallel \mathcal{M}^{\sigma\lambda} \parallel \Psi^{J_i\pi_i}(E) > \right|^2$
- Final (bound-state) and initial (scattering state) wave functions are determined in the R-matrix theory.
- For both wave functions:

internal region

$$\Psi_{\text{int}} = \sum_i f(R_i)\Phi(R_i)$$

External region:

$$\Psi_{\text{ext}} = \phi_1\phi_2(I_\ell(r) - U_\ell O_\ell(r))$$

Step 1: Calculation of the matrix elements between SD $< \Phi^{J_f\pi_f}(R_i) \parallel \mathcal{M}^{\sigma\lambda} \parallel \Phi^{J_i\pi_i}(R_j) >$

Step 2: Correction over the external region: $< \Phi^{J_f\pi_f}(R_i) \parallel \mathcal{M}^{\sigma\lambda} \parallel \Phi^{J_i\pi_i}(R_j) >_{int}$

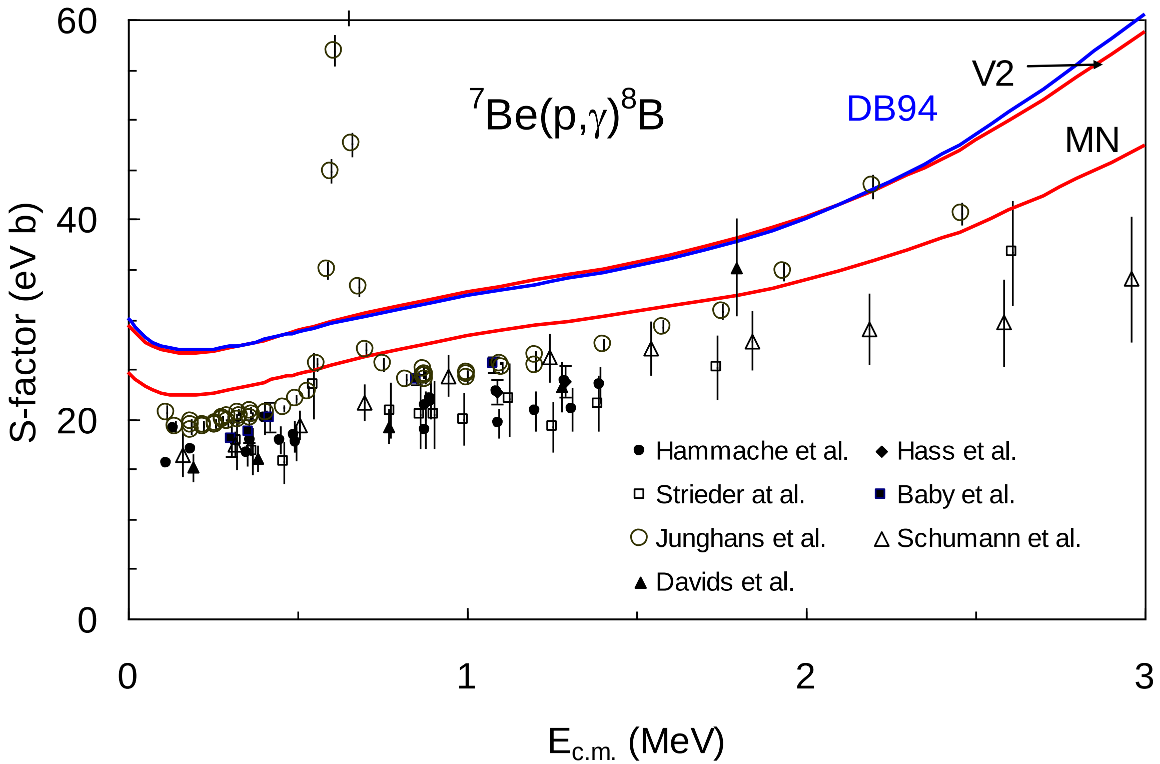
Step 3: Combination of SD:

$$< \Psi^{J_f\pi_f} \parallel \mathcal{M}^{\sigma\lambda} \parallel \Psi^{J_i\pi_i}(E) >_{int} = \sum_{ij} f^{J_f\pi_f}(R_i) f^{J_i\pi_i}(R_j) < \Phi^{J_f\pi_f}(R_i) \parallel \mathcal{M}^{\sigma\lambda} \parallel \Phi^{J_i\pi_i}(R_j) >_{int}$$

Step 4: calculation of the external contribution $< \Psi^{J_f\pi_f} \parallel \mathcal{M}^{\sigma\lambda} \parallel \Psi^{J_i\pi_i}(E) >_{ext}$

Step 5: summation (insensitive to the channel radius!)

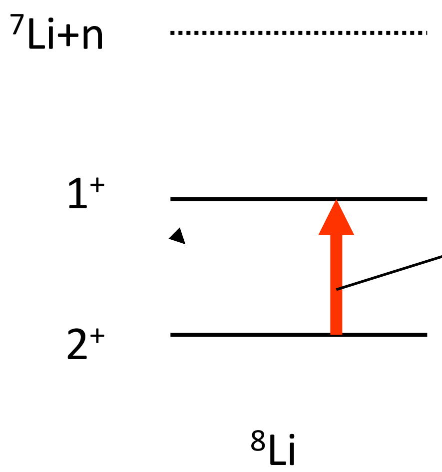
$^7Be(p,\gamma)^8B$ S factor



- Low energies (E<100 keV): energy dependence given by the Coulomb functions
- 2 NN interactions (MN, V2): → the sensitivity can be evaluated
- Overestimation: due to the ^8B ground state (cluster approximation)

9. Some applications: $^7Be(p,\gamma)^8B$

Related measurement: E2 transition in ^8Li
Mirror nucleus of ^8B: only the Coulomb interaction is different



	GCM	exp
B(M1)	7.0	5.0 ± 1.6
B(E2)	2.1	30 ± 15, 55 ± 15

New physics?

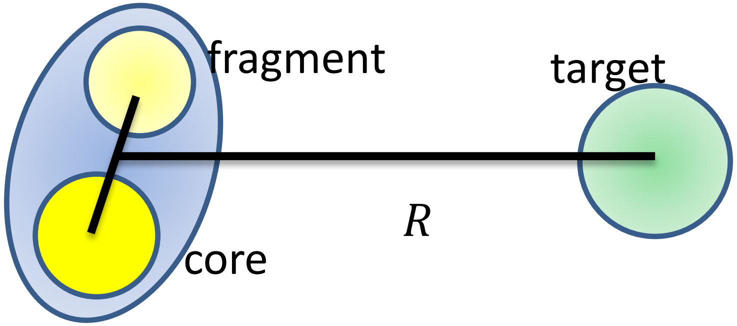
Experimental problem?

→ experiment(s) at USP

B(E2) exp:

- Inelastic cross sections $^8Li(2^+)(X,X)^8Li(1^+)$, with X=^{12}C or Ni (^8Li beam)
- Refs: R.J. Smith *et al.*, Phys. Rev. C43 (1991) 2346
  J.A. Brown *et al.*, Phys. Rev. Lett. 66 (1991) 2452
- Theory: P.D., D. Baye, Phys. Lett. B292 (1992) 235

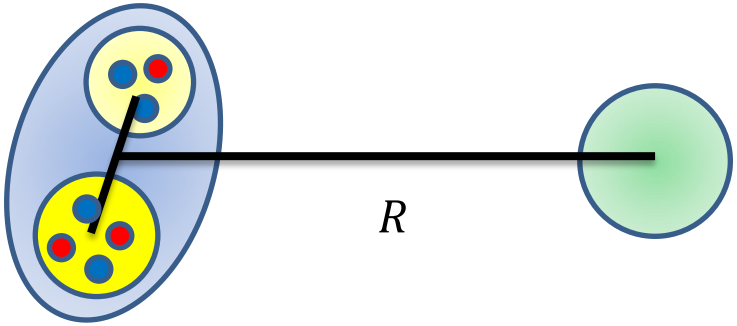
Standard CDCC (2-body projectile)



$$H = H_0(r) - \frac{\hbar^2}{2\mu}\Delta_R + V_{ct}\left(-\frac{A_f}{A}r + R\right) + V_{ft}\left(\frac{A_c}{A}r + R\right)$$

Nucleus-target potentials

Microscopic CDCC (2-cluster projectile)



$$H = H_0(r_1 \dots r_A) - \frac{\hbar^2}{2\mu}\Delta_R + \sum_i v(r_i - R)$$

Nucleon-target potentials

9. Some applications: microscopic CDCC

Application to $^{7}Li+^{208}Pb$

- Data: E_{lab}=27 to 60 MeV (Coulomb barrier ~ 35 MeV)

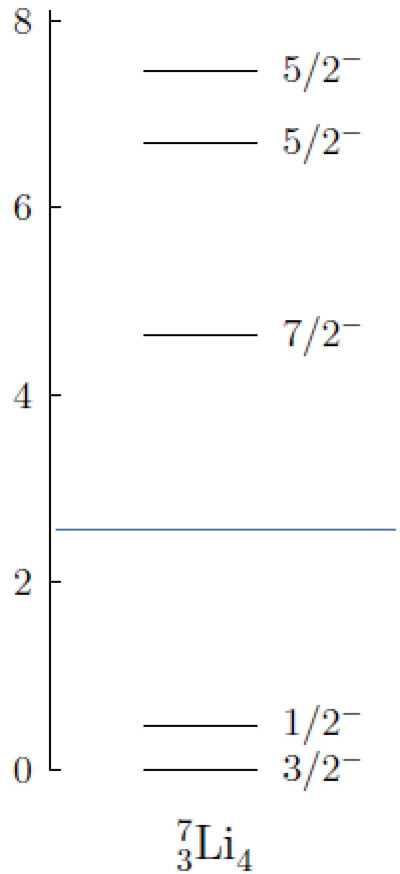
- **Non-microscopic calculation** at 27 MeV:
    - Parkar et al, PRC78 (2008) 021601
    - uses $\alpha-{}^{208}Pb$ and $t-{}^{208}Pb$ potentials renormalized by 0.6!

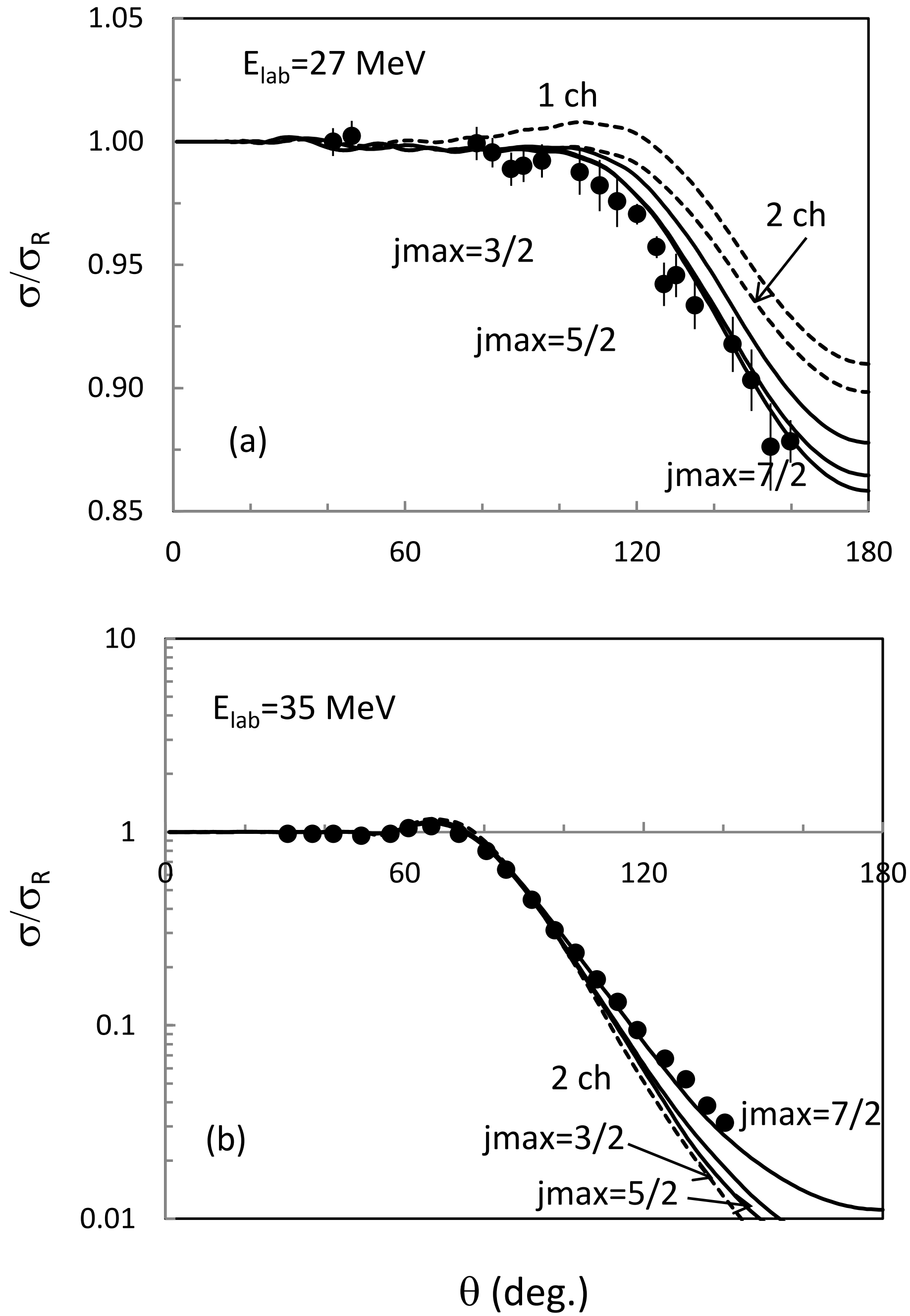
- **Microscopic calculation**
    - ^{7}Li wave functions: include gs,$1/2^-$,$7/2^-$,$5/2^-$ and pseudostates (E>0)
      Nucleon-nucleon potential: Minnesota interaction
      Reproduces $^{7}Li/^{7}Be$, $\alpha+^{3}He$ scattering, $^{3}He(\alpha,\gamma)^{7}Be$ cross section

    - $n-{}^{208}Pb$ potential:
      local potential of Koning-Delaroche (Nucl. Phys. A 713 (2003) 231)
    - $p-{}^{208}Pb$ potential:
      only Coulomb (E_p=27/7 ~4 MeV, Coulomb barrier ~12 MeV)

  ➔ NO PARAMETER

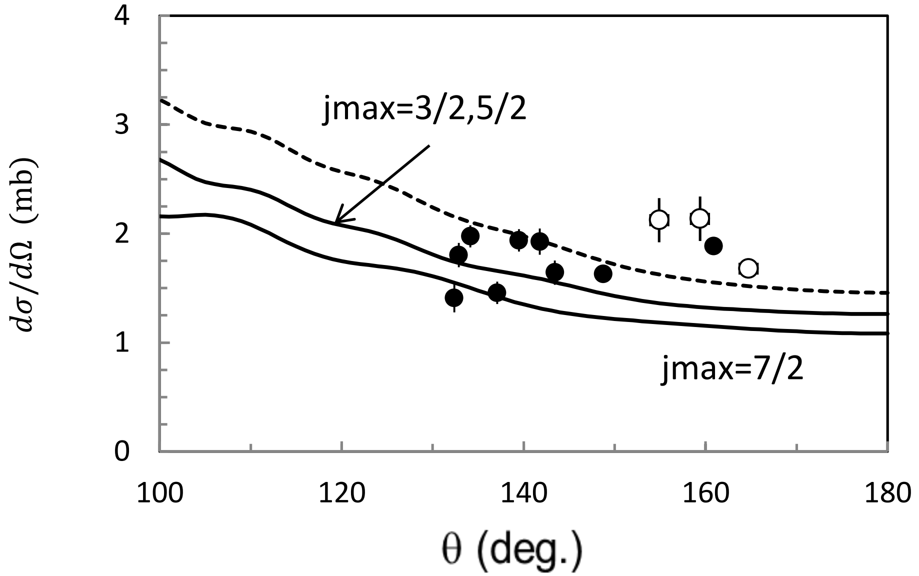
    - Convergence test: single-channel: $^{7}Li(3/2^-)+{}^{208}Pb$
      two channels: $^{7}Li(3/2^-,1/2^-)+{}^{208}Pb$
      multichannel: $^{7}Li(3/2^-,1/2^-,\ldots)+{}^{208}Pb$





$^{7}Li+^{208}Pb$ elastic scattering

$^{7}Li+^{208}Pb$ inelastic scattering



10. Conclusion

- Predictive power of microscopic models

- Inputs
  - NN interaction
  - Cluster assumption

- Spectroscopy and reactions

- Two « families » of microscopic models
  - Include cluster approximation : simplifies the calculations
    → access to reactions, to p-shell and sd-shell nuclei
    → resonant states
  - Do not use the cluster approximation
    → use of realistic interactions

- GCM/RGM wave functions can be used in CDCC calculations