Microscopic folding potentials for inelastic reactions

J. A. Lay*
Dipartimento di Fisica e Astronomia, Università di Padova, IT-35131 Padova, Italy and
Instituto Nazionale di Fisica Nucleare, Sezione di Padova, IT-35131 Padova, Italy
(Dated: October 21, 2013)

We investigate the use of effective nucleon-nucleon (NN) interactions to describe inelastic scattering reactions in a semi-microscopic coupled-channels (CC) formalism. The potentials needed for the calculation of inelastic cross sections are constructed by folding the NN interaction with transition densities calculated in an appropriate structure model. Under certain assumptions (e.g. rotor model), these transition densities can be related to the derivative of the ground state density. This approximation is tested for proton inelastic scattering on ^{10}Be.

PACS numbers: 24.10.Eq, 25.10.+s, 25.45.De

I. INTRODUCTION

One of the key ingredients in nuclear reactions studied within the optical model (OM) or the coupled-channels formalism is the correct evaluation of the projectile-target interaction [1]. This implies the choice of a certain strength and shape of the dependence with the relative coordinate between particle and target, broadly known as optical potentials and defined in terms of different optical parameters. These parameters are usually fixed according to the properties and sizes of the components of the reaction, taken from the analysis of similar reactions with nuclei with similar mass or charge, or directly from global parameterizations like Perey and Perey [2], Hinterberger *et al.* [3], Perrin *et al.* [4], and Daehnick *et al.* [5] One may also adjust these parameters to reproduce the experimental cross section and then infer physical properties of the constituents.

An alternative to the problem of playing with so many parameters is the use of folding potentials [6]. It is possible to obtain nucleon-nucleus and nucleus-nucleus potentials by folding an appropriate effective NN interaction with the corresponding densities. This procedure is widely used for elastic scattering [7–9]. We can also find examples of folding potentials in inelastic scattering [10–13] and even as part of more complex continuum-discretized couple-channels (CDCC) calculations [14–18]. For elastic potentials only the central density for the ground state of the nucleus is needed whereas, for inelastic scattering, the calculation involves the wave function for the excited state and the transition density between this state and the ground state.

This fact increases the difficulty of the calculation and discourages its general use by the community. On the other hand, new developments in reactions with halo nuclei concerning *core* excitations and their implications [19] rely on a good description of the inelastic scattering of the *core* with the target. The usual option in this case is to assume the *core* to be a rotor or a vibrator [20], although again we should play with a large amount of parameters. In this case, it will be useful as well to relate the results of using transition densities with this easier and more common prescription. Therefore, it is our aim to study the use of folding potentials for these purposes as well as possible simplifications like the particle-rotor prescription. We hope it will lead to spread the use of this tool.

In this work, we study the use of folding potentials in the elastic and inelastic scattering of protons on ^{10}Be at 59 MeV using the NN interaction of Jeukenne, Lejeune and Mahaux [21] and the densities calculated microscopically using the antisymmetrized molecular dynamics (AMD) method [22] kindly supplied by Prof. Y. Kanada-En'yo. The cross sections will be calculated within the CC formalism and compared with the available experimental data [23, 24].

The paper is structured as follows. In Sec II we explain how to construct the folding potential from transition densities using a density dependent NN interaction. In Sec. III we apply the method to proton elastic and inelastic scattering on ^{10}Be. Finally, in Sec. IV the main conclusions of this work are summarized.

II. FOLDING MODEL FOR THE PROJECTILE-TARGET INTERACTION

We will perform CC calculations for both elastic and inelastic cross sections of a proton on a nucleus. In order to generate the corresponding potential, we start from the general convolution of an effective in-medium NN interaction with the density of the projectile:

$$V_{vc}(\vec{r},\xi) = \int \rho(\vec{r'},\xi) v_{nn}(\vec{r}-\vec{r'}) d\vec{r'}. \qquad (1)$$

where v_{nn} is the effective NN interaction and $\rho(\vec{r'},\xi)$ the density operator, defined as usual as

$$\rho(\vec{r},\xi) = \sum_{i=1}^{A} \delta(\vec{r}-\vec{r}_i) \qquad (2)$$

* lay@pd.infn.it

This is conveniently expanded in multipoles as

$$\rho(\vec{r'},\xi) = \sum_{\lambda\mu} \rho_{\lambda\mu}(r',\xi) Y_{\lambda\mu}(\hat{r'}). \quad (3)$$

Note that, in the spherical case, $\rho(\vec{r'}) = \rho(r')$, and $V_{vc}(r)$ results a central potential, so it provides only the $\lambda = 0$ term in a multipole expansion of the potential. In this case, $\rho(\vec{r'},\xi)$ contains also non-central terms that will give rise to transition potentials with $\lambda > 0$. For the inelastic cross section, we will need at least a term with $\lambda = 2$ since we consider an excitation between a 0^+ ground state and a 2^+ excited state of the target.

One requires the reduced matrix elements of the V_{vc} interaction between different states of the projectile included in the calculation. In the folding scheme, these will be related to the reduced matrix elements of the density operator, i.e.

$$\begin{aligned}\langle I\nu|\rho(\vec{r})|I'\nu'\rangle &= \langle\phi_{I\nu}(\xi)|\sum_{i=1}^{A}\delta(\vec{r}-\vec{r}_i)|\phi_{I\nu'}(\xi)\rangle \\ &= \sum_{\lambda,\mu}\langle I'\nu'\lambda\mu|I\nu\rangle\rho_{\lambda,I\to I'}(r)Y^*_{\lambda\mu}(\hat{r}) \quad (4)\end{aligned}$$

where $\rho_{\lambda,I\to I'}(r)$ correspond to the reduced matrix elements

$$\rho_{\lambda,I\to I'}(r) \equiv \langle I'||\rho_\lambda||I\rangle\,. \quad (5)$$

Our convention for reduced matrix elements is that of Brink and Satchler [25] so that the inverse densities are related as $\sqrt{2I'+1}\langle I'||\rho_\lambda||I\rangle = \sqrt{2I+1}\langle I||\rho_\lambda||I'\rangle$.

The density operator can be analogously defined for protons and neutrons ($\rho^{(p)}$ and $\rho^{(n)}$), in which case the sum in Eq. (2) runs over protons or neutrons, respectively. The corresponding monopole transition densities are normalized as

$$\int \rho^{(p)}_{0,I\to I}(r)Y_{00}(\hat{r})d\vec{r} = Z, \quad (6)$$

$$\int \rho^{(n)}_{0,I\to I}(r)Y_{00}(\hat{r})d\vec{r} = N, \quad (7)$$

For the proton case, the multipole terms are constrained by the electric transition probabilities, i.e.:

$$\mathcal{B}(E\lambda, I\to I') = \frac{2I'+1}{2I+1}e^2\left|\int \rho^p_{\lambda,I\to I'}(r)r^{\lambda+2}dr\right|^2 \quad (8)$$

In this work, the required transition densities are obtained from antisymmetrized molecular dynamics (AMD) [26, 27] calculations. AMD wavefunctions are formed from Slater determinants of single-nucleon Gaussian wavefunctions. The energy of the system is calculated considering effective nucleon-nucleon interactions. The ground state is obtained with a variational method and the excited states, applying the same method for an AMD function orthogonal to those with lower energy. Although no initial cluster structure is assumed, it has been shown that, for Be isotopes, the structure of the low-lying states obtained within this method can be described as two alpha clusters with the remaining neutrons orbiting around the two alphas as proposed by Von Oertzen [28]. Actually, the method has been proved to be very useful to understand the level structure and deformation of Be and B isotopes [22].

Following [6], the central part of the effective nucleon-nucleon interaction (v_{nn}) is decomposed in terms of the total spin (S) and isospin (T) of the colliding pair but, for simplicity, only the $S = 0$ terms are considered,

$$v_{nn}(s) = v_{00}(s) + v_{01}(s)\vec{\tau'}\cdot\vec{\tau}\,, \quad (9)$$

where v_{ST} are the expansion terms and τ is the isospin operator. Attending to the isospin dependence, the v_{00} and v_{01} terms are called, respectively, isoscalar and isovectorial parts. The radial forms $v_{0T}(s)$ are taken from the work of Jeukenne-Lejeune-Mahaux (JLM) [21]

$$\begin{aligned}v_{0T}(s,\rho,E) = {}& \lambda_v V_T(\rho,E)(t_v\sqrt{\pi})^{-3}\exp(-s^2/t_v^2) \\ & + i\lambda_w W_T(\rho,E)(t_w\sqrt{\pi})^{-3}\exp(-s^2/t_w^2). \quad (10)\end{aligned}$$

The strengths of the real and imaginary potentials, V_T and W_T, depend on the density ρ, and the nucleon-nucleon relative energy E. This energy is corrected by an estimation of the Coulomb energy [21]. On the other hand, normalization factors, $\lambda_{v,w}$, and the effective range of the Gaussian form factor, $t_{v,w}$, are adjustable parameters with typical values between 0.8 and 1.2 for $\lambda_{v,w}$, and between 1.2 and 1.4 for $t_{v,w}$. This interaction has been found to reproduce satisfactorily the elastic and inelastic experimental cross sections in the intermediate energy region for light halo nuclei [12, 13].

In order to evaluate eq. (1) we should also expand the interaction in multipoles as we did for the density:

$$v_{0T}(|\vec{r}-\vec{r'}|,\rho,E) = \sum_\ell v^{(\ell)}_{0T}(r,r')Y_\ell(\hat{r})\cdot Y_\ell(\hat{r'})\,. \quad (11)$$

In the test cases considered in this work, the projectile is a proton, in which case the resulting potential can be expressed in terms of the corresponding proton and neutron transition densities as [12]:

$$\begin{aligned}\langle I||V_\lambda(r,\vec{\xi})||I'\rangle = {}& \int r^2dr'\Big\{v^{(\lambda)}_{00}(r,r') \\ & \times\left[\rho^{(p)}_{\lambda,I\to I'}(r)+\rho^{(n)}_{\lambda,I\to I'}(r)\right] \\ & + v^{(\lambda)}_{01}(r,r') \\ & \times\left[\rho^{(p)}_{\lambda,I\to I'}(r')-\rho^{(n)}_{\lambda,I\to I'}(r')\right]\Big\}\,. \quad (12)\end{aligned}$$

Note that, if the projectile is a neutron, the sign of the isovectorial term changes.

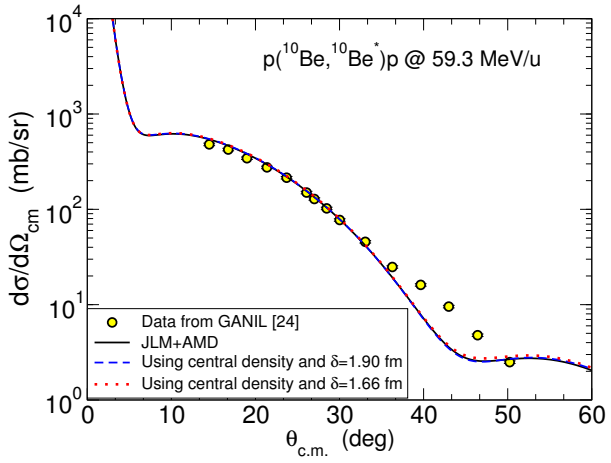

FIG. 1. Angular distribution of the elastic cross section of protons on ^{10}Be at 59 MeV for the three sets of potentials described in the text. We compare with the data from [24]

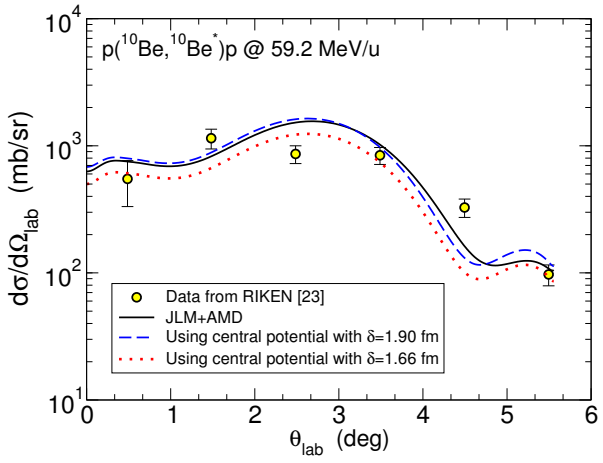

FIG. 2. Angular distribution of the inelastic cross section of protons on ^{10}Be at 59 MeV for the three sets of potentials described in the text. We compare with the data from [23]

III. APPLICATION TO INELASTIC SCATTERING

We perform a CC calculation for the elastic and inelastic scattering of protons on ^{10}Be at 59 MeV using the potentials calculated as described in the previous section. The states considered for the inelastic scattering are the 0^+ ground state and the 2^+ first excited state of ^{10}Be. AMD densities for ^{10}Be from Y. Kanada-En'yo [26] together with the JLM interaction [21] are used to construct the corresponding projectile-target potentials. Concerning the parameters involved in the interaction, we use $t_{v,w} = 1.2$ as suggested in the original work [21] and a standard normalization for the real and imaginary part, which is $\lambda_v = 1.0$ and $\lambda_w = 0.8$ [24]. Different parameters as well as different normalizations for real and imaginary parts are used in previous works for the same reactions [13, 24] in order to fit the data. However, our aim is to check how good is the agreement with the data for standard values of the parameters, so that we will keep these values.

For the numerical calculation of the potential we have developed the program MIFOLD. During this development it has been a great help to compare with the results of the code MINC from Prof. M. Takashina [29]. For the CC calculation we used the code FRESCO [30]. The results are shown in Figs. 1 and 2 for the elastic and inelastic cross section respectively (solid lines). In both cases there is a good agreement with the data.

In order to simplify the present calculation, we will consider a situation where only the ground state density is available. In this case, we can estimate the transition density according to rotor or vibrator models [20] which are reasonable approximations for many nuclei with a first excited state 2^+. Within this model, the transition densities are considered to be proportional to the central potential:

$$\rho_{2,0^+\to 2^+} \approx \langle 2^+||\hat{\delta}_2||0^+\rangle \frac{d\rho_{0,0^+\to 0^+}}{dr}, \qquad (13)$$

where the matrix element $\langle 2^+||\hat{\delta}_2||0^+\rangle$ is related, within the rotor model, to the deformation length as:

$$\langle I'||\hat{\delta}_\lambda||I\rangle = \langle I'K\lambda 0|IK\rangle\delta_\lambda = \langle I'K\lambda 0|IK\rangle\beta_\lambda R, \qquad (14)$$

where K is the projection of the angular momentum along the symmetry axis that characterizes the rotational band, so that for $0^+ \to 2^+$ transition we have $K = 0$.

Comparing this approximated $\rho_{2,0^+\to 2^+}$ with the corresponding AMD density we find a deformation length of $\delta_2 = 1.90$ fm, larger than the usual values in particle-rotor calculations, e. g. $\delta_2 = 1.66$ fm [31, 32]. Using this density we perform again the calculation of the elastic and inelastic cross sections (dashed and dotted lines in Figs. 1 and 2 for the calculated deformed length and the one used in [31, 32] respectively). Both deformation lengths lead to reasonable results since larger differences can be found altering the values of the JLM interaction [13]. The two values are also consistent with the deformation length obtained in [23] by fitting the experimental inelastic data with different particle-rotor potentials, $\delta_2 = 1.80 \pm 0.25$ fm. Larger values are found in previous analysis of inelastic cross sections at lower energies, $1.84 - 1.99$ fm [33], but still consistent with the calculated deformed length $\delta_2 = 1.90$ fm.

IV. SUMMARY AND CONCLUSIONS

We have shown how to construct folding potentials for the elastic and inelastic scattering of nucleons on a nucleus. We have applied this method to the case of protons

on ^{10}Be at 59 MeV considering the ground state and the first excited state of this nucleus. Using the JLM effective NN interaction and the AMD densities for ^{10}Be, we obtain a good agreement with the available data.

In order to avoid the difficulty of computing transition densities for the excited state, we have repeated the calculations using only the central density and its derivative following the prescription of collective models. In this case, the deformation length is a free parameter that we can set by comparing with the original transition density. This gives a value of $\delta_2 = 1.90$ fm, larger than the values previously used for particle-rotor calculations, $\delta_2 = 1.66$ fm, but consistent with the value obtained from previous analyses of inelastic scattering experiments [23, 33]. Nevertheless, with the three sets of potentials, using the original transition density and these two values of δ multiplied times the derivative of the central density, similar cross sections are obtained, all of them consistent with the data.

Therefore, we conclude that through the derivative of the density it is possible to obtain reasonable results for the inelastic scattering. The analysis of the data can also give a preliminary value for the nuclear deformation for those cases where only the ground state is well known. The only ingredient should be a good description of the ground state density as obtained here thanks to the AMD calculations by Prof. Y. Kanada-En'yo. This central density can be obtained by a variety of models, such as Hartree-Fock or beyond-mean-field approaches. However, it is obvious that, for those nuclei where we can obtain them, the best option is to perform the analysis using the transition densities together with an appropriate NN interaction like JLM.

ACKNOWLEDGMENTS

This work was developed as a project for the 6^{th} TALENT course carried out at GANIL under the supervision of Prof. A. M. Moro and J. A. Tostevin. We acknowledge Prof. Y. Kanada-En'yo for the AMD densities and useful discussion and Prof. M. Takashina for the code MINC. J.A.L. thanks the organization of the 6^{th} TALENT course for selecting him to participate in the course.

[1] G. R. Satchler, *Direct Nuclear Reactions* (Clarendon Press, Oxford, 1983).
[2] C. M. Perey and F. G. Perey, Phys. Rev., **132**, 755 (1963).
[3] F. Hinterberger, G. Mairle, U. Schmidt-Roht, G. J. Wagner, and P. Turek, Nucl. Phys. A, **111**, 265 (1968).
[4] G. Perrin, N. V. Sen, J. Arvieux, R. Darves-Blanc, J. Durand, A. Fiore, J. Gondrand, F. Merchez, and C. Perrin, Nucl. Phys. A, **282**, 221 (1977).
[5] W. W. Daehnick, J. D. Childs, and Z. Vrcelj, Phys. Rev. C, **21**, 2253 (1980).
[6] G. Satchler and W. Love, Phys. Rep., **55**, 183 (1979).
[7] D. Y. Pang, Y. L. Ye, and F. R. Xu, Phys. Rev. C, **83**, 064619 (2011).
[8] W. Y. So, T. Udagawa, K. S. Kim, S. W. Hong, and B. T. Kim, Phys. Rev. C, **76**, 024613 (2007).
[9] D. T. Khoa, G. R. Satchler, and N. D. Thuy, Phys. Rev. C, **65**, 024611 (2002).
[10] D. T. Khoa and G. Satchler, Nuclear Physics A, **668**, 3 (2000).
[11] Y. Sakuragi, Y. Maenaka, and T. Furumoto, J. Phys.: Conf. Ser., **111**, 012019 (2008).
[12] M. Takashina, Y. Kanada-En'yo, and Y. Sakuragi, Phys. Rev. C, **71**, 054602 (2005).
[13] M. Takashina and Y. Kanada-En'yo, Phys. Rev. C, **77**, 014604 (2008).
[14] Y. Y. Yang *et al.*, Phys. Rev. C, **87**, 044613 (2013).
[15] K. C. C. Pires *et al.*, Phys. Rev. C, **83**, 064603 (2011).
[16] A. Barioni *et al.*, Phys. Rev. C, **80**, 034617 (2009).
[17] M. Cubero *et al.*, Phys. Rev. Lett., **109**, 262701 (2012).
[18] J. P. Fernández-García *et al.*, Phys. Rev. Lett., **110**, 142701 (2013).
[19] A. M. Moro and J. A. Lay, Phys. Rev. Lett., **109**, 232502 (2012).
[20] A. Bohr and B. Mottelson, *Nuclear Structure*, New York, W. A. Benjamin ed. (1969).
[21] J. P. Jeukenne, A. Lejeune, and C. Mahaux, Phys. Rev. C, **16**, 80 (1977).
[22] Y. Kanada-Enyo, H. Horiuchi, and A. Ono, Phys.Rev., **C52**, 628 (1995); Y. Kanada-Enyo and H. Horiuchi, *ibid.*, **C52**, 647 (1995).
[23] H. Iwasaki *et al.*, Physics Letters B, **481**, 7 (2000).
[24] M. Cortina-Gil *et al.*, Physics Letters B, **401**, 9 (1997).
[25] D. M. Brink and G. R. Satchler, *Angular Momentum* (Clarendon, Oxford, 1968).
[26] Y. Kanada-En'yo, H. Horiuchi, and A. Doté, Phys. Rev. C, **60**, 064304 (1999).
[27] Y. Kanada-En'yo, Phys. Rev. C, **71**, 014310 (2005).
[28] W. von Oertzen, Z. Phys. A, **354**, 37 (1996).
[29] M. Takashina, Single-folding model code MINC. Private Comunication.
[30] I. Thompson, Comp. Phys. Rep. **7**, 167 (1988).
[31] J. A. Lay, A. M. Moro, J. M. Arias, and J. Gómez-Camacho, Phys. Rev. C, **85**, 054618 (2012).
[32] F. M. Nunes, J. A. Christley, I. J. Thompson, R. C. Johnson, and V. D. Efros, Nucl. Phys. A, **609**, 43 (1996).
[33] D. L. Auton, Nuclear Physics A, **157**, 305 (1970).