

ISOMERS Pb - Po - Rn - Fr...

- 1] Technicalities: lifetimes/widths/strengths..
 - 2] Building blocks...
 - 3] Signatures of structure: seniority isomers, K-isomers
 - 4] Triple shape coexistence - ^{188}Pb example
-

Four Case Studies

Case I: ^{209}Fr - maybe

Case II: ^{212}Rn - E3 transitions; signatures
- Empirical Shell Model View
- Deformed Independent Particle Model View

Case III: $^{190,192,194,194}\text{Pb}$ - 11^- isomers; E3 strengths
- oblate deformation

Case IV: ^{189}Pb - $\text{vi}_{13/2}^{-3}$ isomer ?
- residual interactions/ non-maximal coupling

τ ... and the meaning of life...

$$\lambda \geq I_i - I_f \quad \begin{array}{c} I_i \\ \text{-----} \\ I_f \end{array} \quad \begin{array}{c} \updownarrow \\ \Delta E \end{array}$$

$$\tau \sim 1 / \{ |\langle f | T_\lambda | i \rangle|^2 \times \Delta E^{(2\lambda+1)} \} \quad \text{***von Weizsacker 1936}$$

structure dependent

partly structure dependent
partly random

difficult to calculate for
forbidden transitions

difficult to predict ΔE
accurately

**low energy
small matrix element
high multipolarity...**

nowhere to go...

Play School

τ (not $T_{1/2}$) - the "natural" unit

$$\Gamma.\tau = 0.658 \times 10^{-15} \text{ eV.s}$$

$$\begin{aligned}\Gamma &= \Gamma_{\gamma_1} + \Gamma_{e_1} + \Gamma_{\gamma_2} + \Gamma_{e_2} + \dots \\ &= (1 + \alpha_1^T)\Gamma_{\gamma_1} + (1 + \alpha_2^T)\Gamma_{\gamma_2} + \dots\end{aligned}$$

Weisskopf unit (eV)

$$\Gamma_W(E1) = 6.748 \times 10^{-2} A^{2/3} E_{\gamma}^3$$

$$\Gamma_W(E2) = 4.792 \times 10^{-8} A^{4/3} E_{\gamma}^5$$

$$\Gamma_W(M1) = 2.072 \times 10^{-2} E_{\gamma}^3$$

$$\Gamma_W(M2) = 1.472 \times 10^{-8} A^{2/3} E_{\gamma}^5$$

$$\Gamma_W(E3) = 2.333 \times 10^{-14} A^2 E_{\gamma}^7$$

Play School

τ (not $T_{1/2}$) - the "natural" unit

$$\Gamma \cdot \tau = 0.658 \times 10^{-15} \text{ eV} \cdot \text{s}$$

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typical lifetimes:(Pb 500 keV)

10^{-6} W.u.	E1	23 ns
1 W.u.	E2	0.4 ns
0.1 W.u.	M1	0.002 ns
< 1 W.u.	M2	> 32 ns
10 W.u.	E3	8.6 μs

a long lifetime does
not imply hindrance

How

Yrast Line

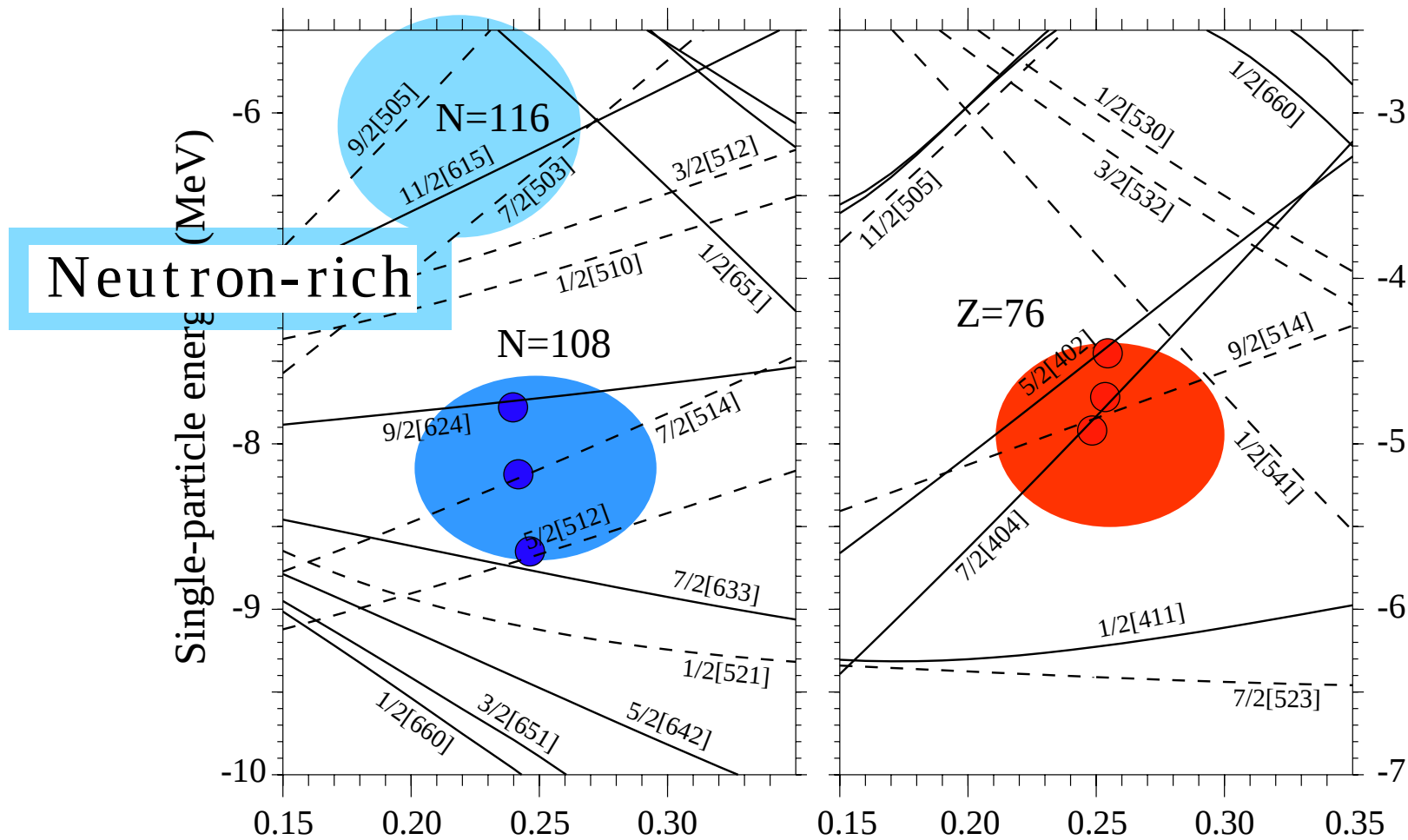
*Residual
spin-spin
 j or K*

*Non-uniform
levels -shell gaps
spherical or deformed*

*High- j or
High- K*



$Z \sim 76$; $N \sim 104$ favoured region for high-K



$$K = v |\Omega_1 + \Omega_2 + \Omega_3 \dots| + \pi |\Omega_1 + \Omega_2 + \Omega_3 \dots|$$

$$\sim 9/2 + 7/2 + 5/2 \dots + 7/2 + 9/2 + 5/2 \dots = 21$$

High-K Intrinsic states cost less energy than rotation

Z = 72
N ~ 104

common deformed building blocks

align high- Ω orbitals

$8^- \quad \nu \quad 9/2^+ [624] \times 7/2^- [514]$
 $8^- \quad \pi \quad 9/2^- [514] \times 7/2^+ [404]$

16^+

lower because of residual interaction

^{178}Hf exp.
the 31-year isomer

8^- 1147

8^- 1479

16^+ 2626 keV

cf: 2447 keV

Z = 72
N ~ 104

common deformed building blocks

align high- Ω orbitals

8⁻ ν 9/2⁺[624] x 7/2⁻ [514]
8⁻ π 9/2⁻ [514] x 7/2⁺[404]

16⁺

¹⁷⁸Hf exp.
the 31-year isomer

8⁻ 1147
8⁻ 1479

16⁺ 2626 keV
cf: 2447 keV

Z ~ 82-
N ~ 124

common spherical building blocks

align high-j orbitals

²¹²Po exp.
the 65 sec. isomer

e.g.

8⁺ π h_{9/2}²
10⁺ ν g_{9/2}i_{11/2}

18⁺

8⁺ 1476
10⁺ 1834

18⁺ 3310 keV
cf: 2922 keV

π
f_{7/2}
i_{13/2}
h_{9/2}

82

s_{1/2}
d_{3/2}

j_{15/2}
i_{11/2}
g_{9/2}
126 ν
p_{1/2}
f_{5/2}
p_{3/2}
i_{13/2}
h_{9/2}
f_{7/2}
82

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82

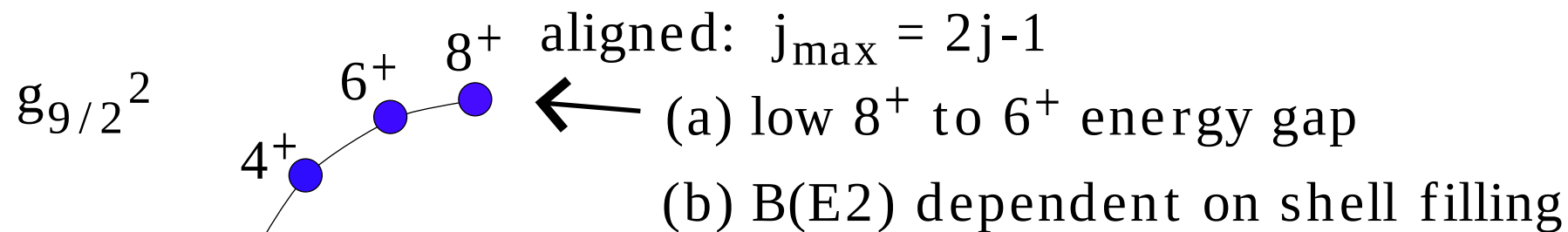
11⁻ π h_{9/2}i_{13/2} , 12⁺ ν i_{13/2}⁻²

many factors...

Low transition energies - low orbitals + favourable interactions, large configuration changes, K-hindrance, etc. etc.

e.g.

j^n multiplets - two (or more) identical particles in single j -shell



"seniority-2" [isolated shell]

$$B(E2) \propto (2\Omega - 2n)^2 \rightarrow 0 \text{ at } n=5$$

[shell degeneracy $2\Omega = 2j+1$]

"quasiparticles" [mixed shells]

$$B(E2) \propto (u^2 - v^2)^2 \times e_{\text{eff}}^2 \rightarrow 0 \text{ at } u=v$$

[occupation prob. u, v]

0^+ anti-aligned;
(additional depression
from pairing)

signature of shells

Zhang et al PRC 62 (2000) 057305: 10^+ and $27/2^-$ isomers

$$m = 11/2 + 9/2 + 7/2 = 27/2$$

$\nu h_{11/2}^3$ 8^+ 10^+

6^+

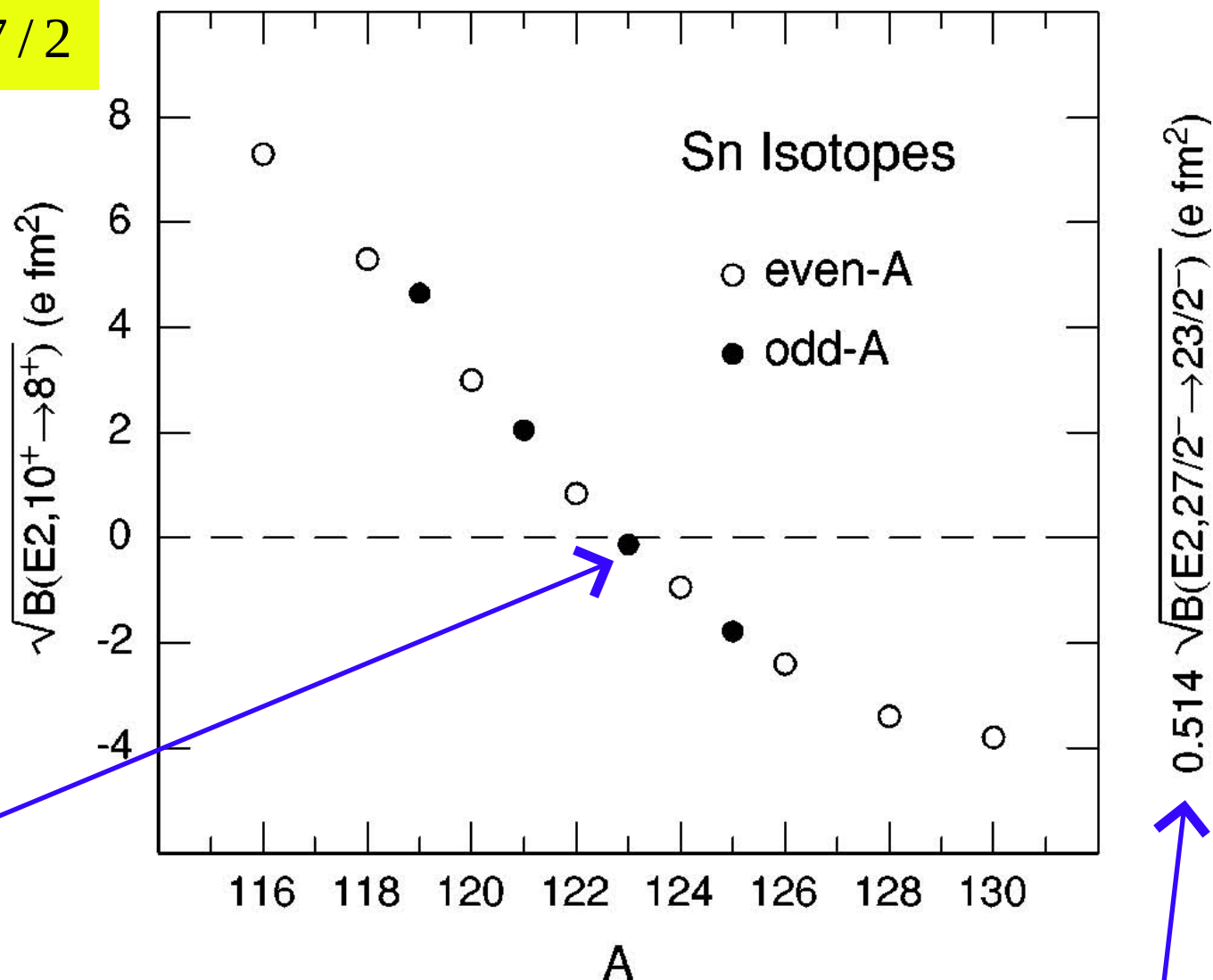
$\nu h_{11/2}^2$

4^+

2^+

0^+

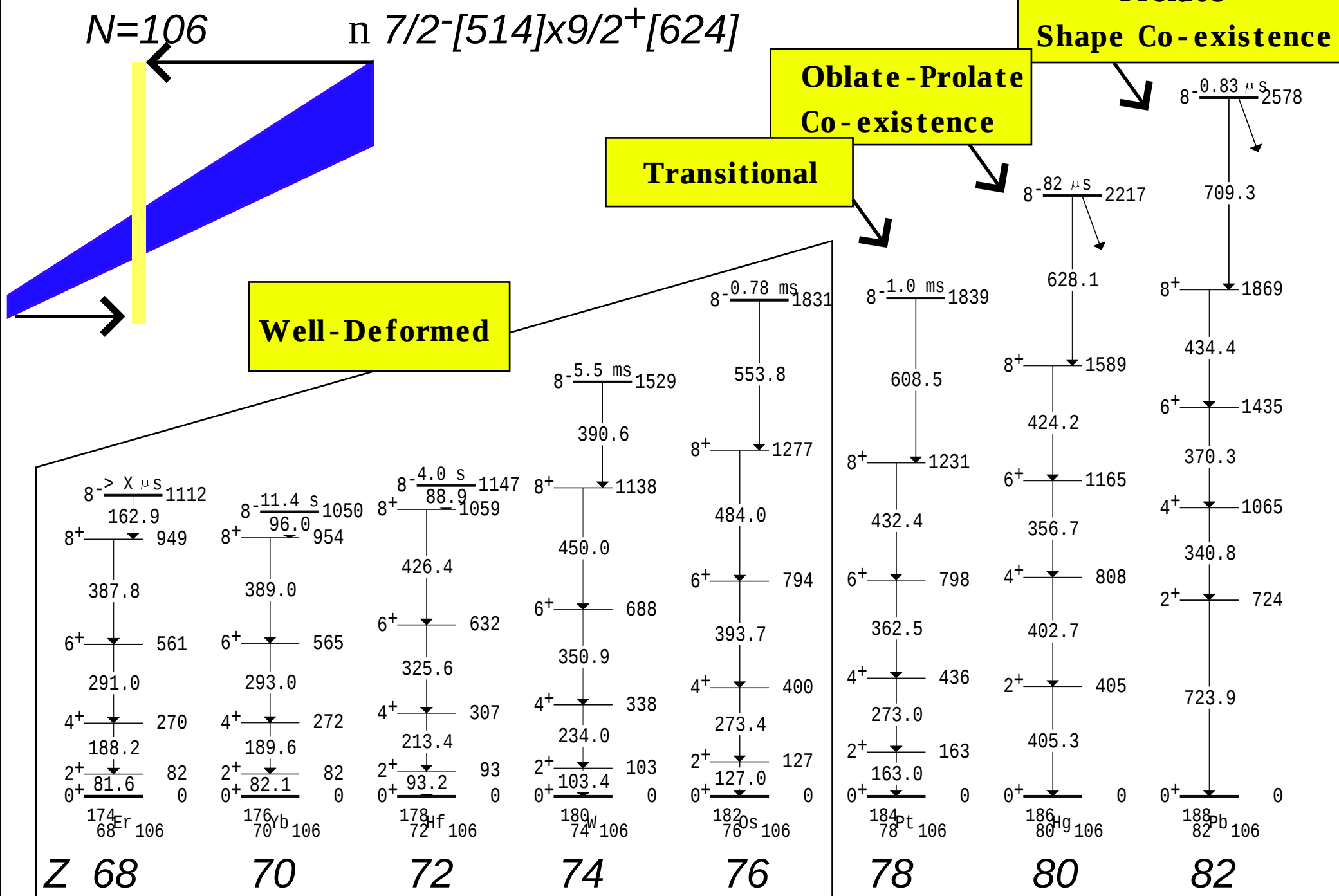
negative sign
assumed



10^+ to 8^+ compared to $27/2^-$ to $23/2^-$
[adjusted for $\nu=3/\nu=2$ geometric factors]

$K^\pi = 8^-$ Isomer; Signature of prolate Deformation

Identification of ^{174}Er & 8- isomer in prolate Well



Triple shape co-existence - Isomers

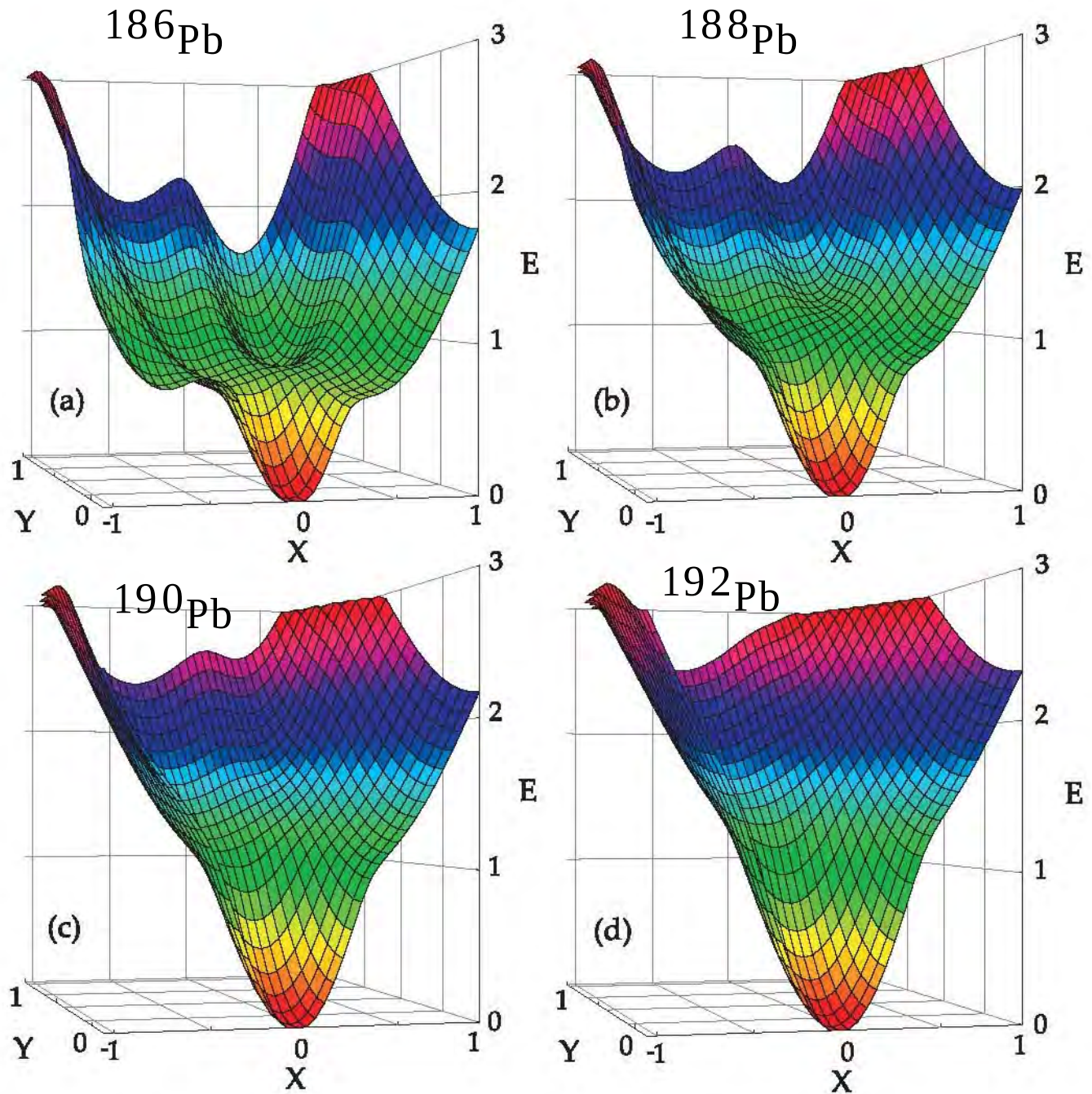
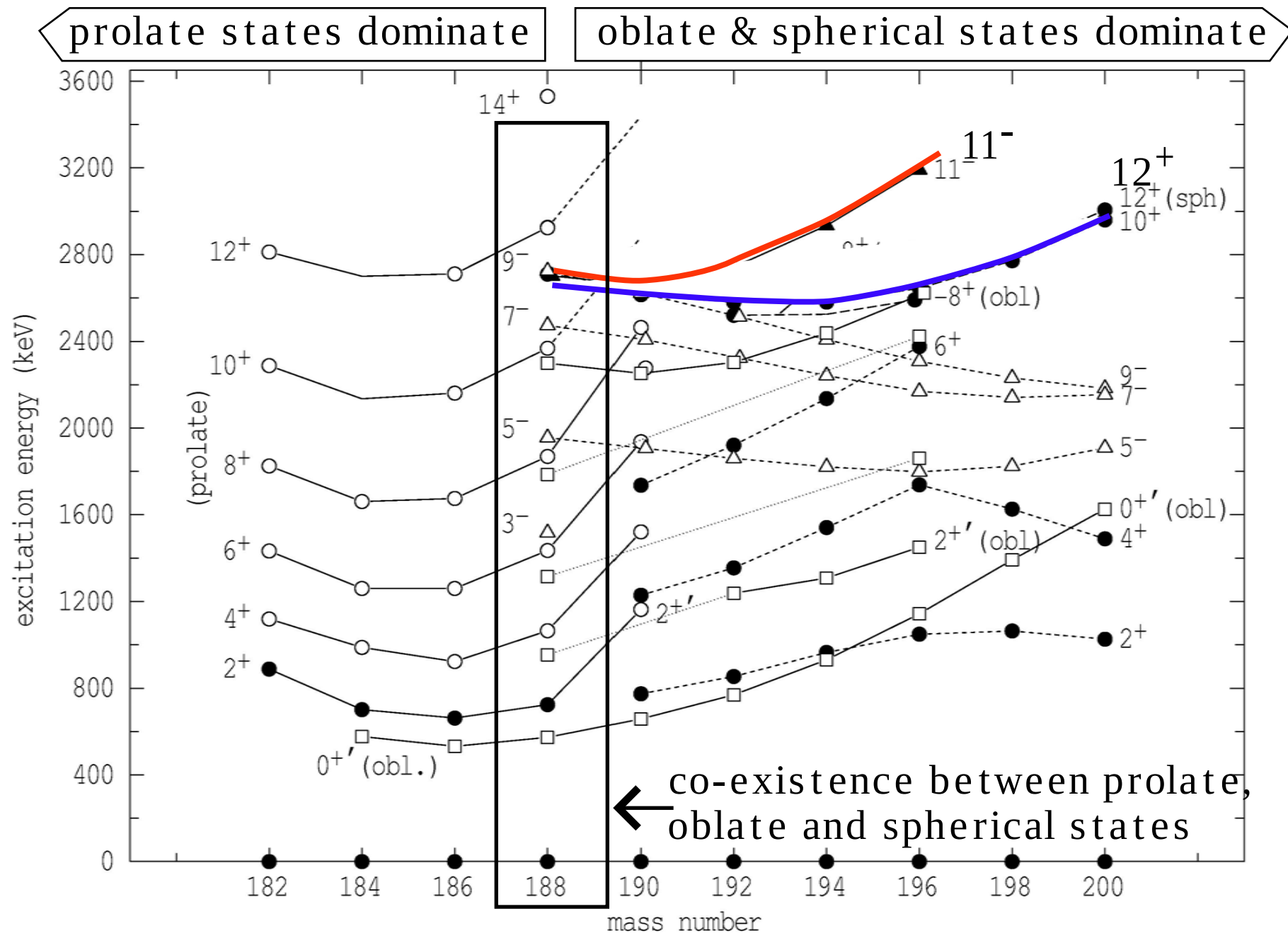


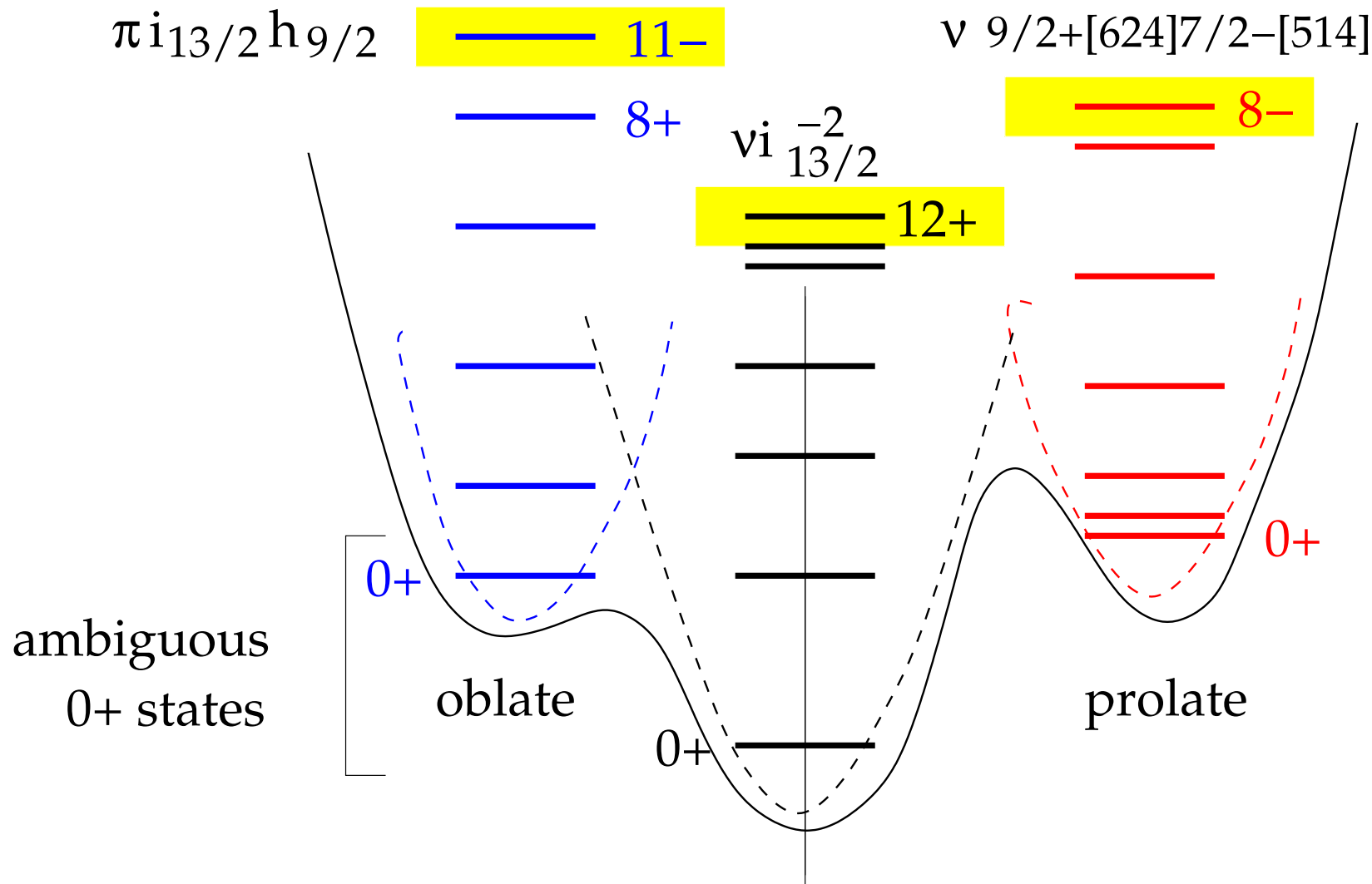
FIGURE 3.1: Energy surfaces for (a) ^{186}Pb , (b) ^{188}Pb , (c) ^{190}Pb , and (d) ^{192}Pb . On the horizontal axes, $X \equiv \beta \sin(\gamma + \pi/6)$, while $Y \equiv \beta \cos(\gamma + \pi/6)$. The vertical axis gives the energy in MeV. The figure is taken from [Fra04] and the parameters used to construct the surfaces were determined in [Fos03].

Pb Systematics



predicted Z=82 Triple Well

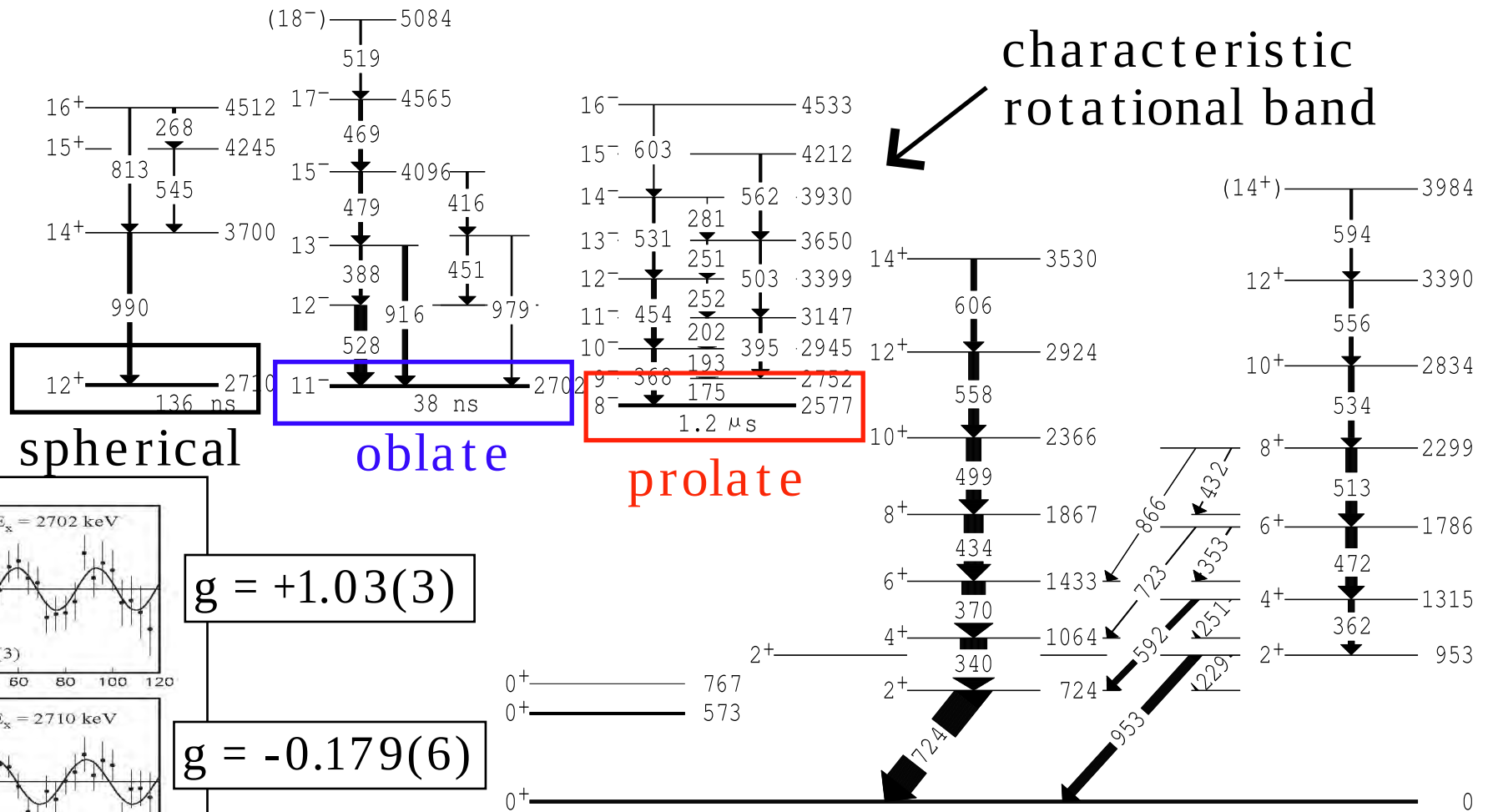
characteristic isomers in each: structure, spin and parity



structures above isomers also distinctive

^{188}Pb spherical/oblate/prolate

GDD et al. , PRC 67, 05130(R) (2003); PRC 69, 054318 (2004)



Ionescu-Bujor et al. PRC 81, 024323 (2010)
 "g-factors of co-existing isomeric states in ^{188}Pb "

CASE - I: ^{209}Fr
[5 valence protons; 4 neutron holes]

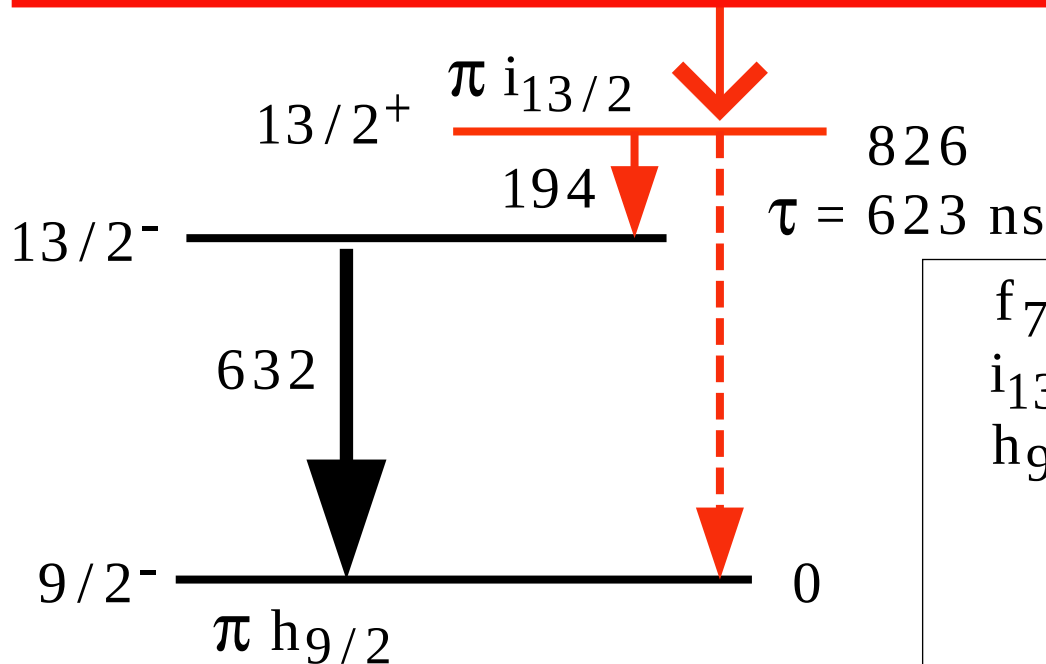
Case I: ^{209}Fr $Z=87$, $N=122$, $i_{13/2}$ proton state?

$$\Gamma(E1,194) = 1 \times 10^{-9} \text{ eV}$$

$$\pi i_{13/2} \text{ to } h_{9/2} \text{ M2} \sim 0.2 \text{ W.u.} \leftarrow$$

$$\Gamma(M2,826) \sim 40 \times 10^{-9} \text{ eV}; \tau_\gamma = 17 \text{ ns}$$

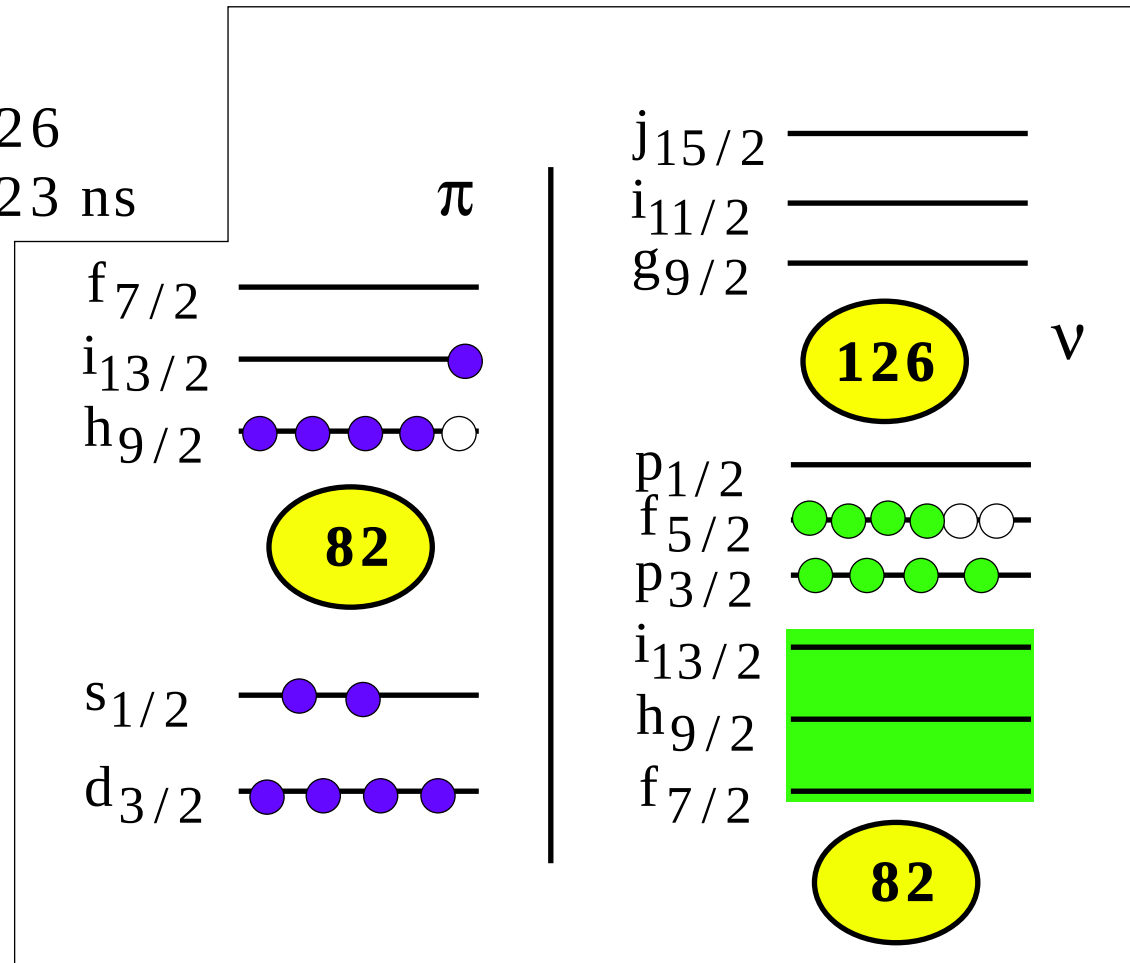
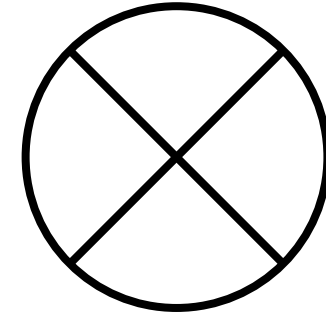
M2 BRANCH SHOULD DOMINATE



^{209}Fr

Meyer et al

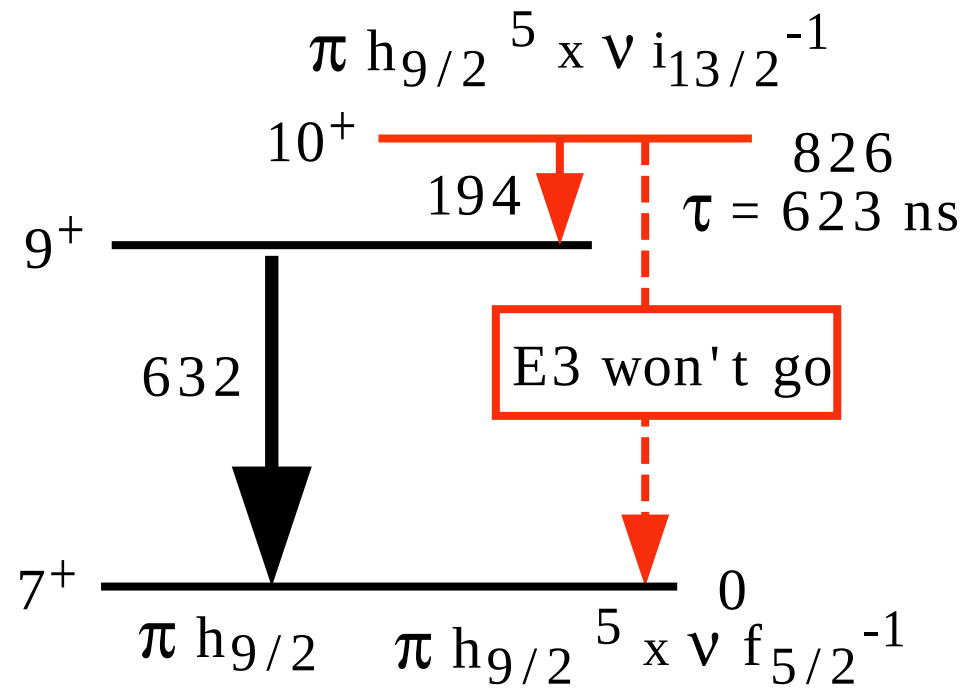
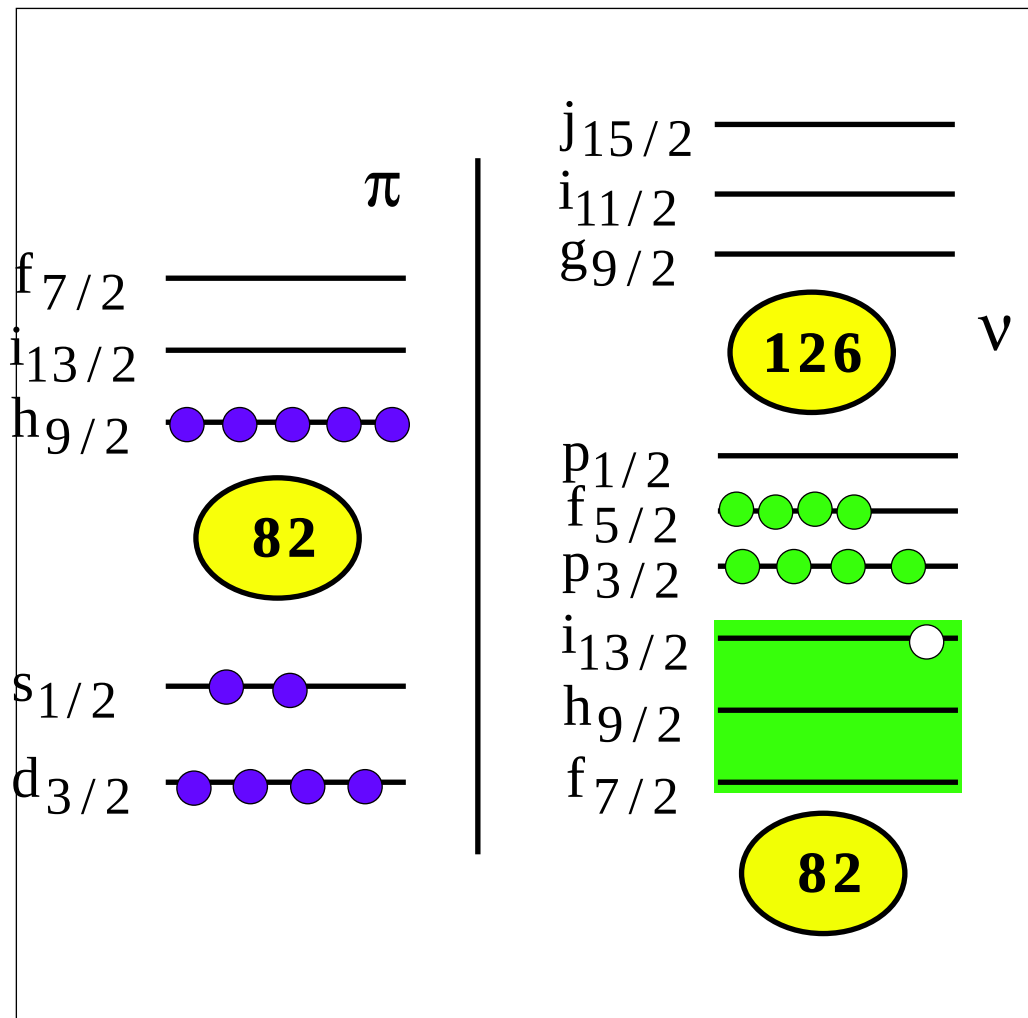
Phys Rev C 73 (2006) 024307



Case I: the drastic solution...

$$\Gamma(E1,194) = 1 \times 10^{-9} \text{ eV}$$

STRUCTURE IS IN ^{208}Fr !!



^{208}Fr !

GDD et al
EPJA 40(2009)127

CASE - II: ^{212}Rn
[4 valence protons; 0 neutron holes
plus neutron core excitations]
Signature E3 Transitions

See Phys Rev C-80-2009-05430
Phys Lett B 662-2008-19

Spherical levels and octupole coupling -E3 signatures

$$|i_{13/2}^{\sim}\rangle = \alpha i_{13/2} + \beta f_{7/2} \otimes 3^- + \gamma \dots$$

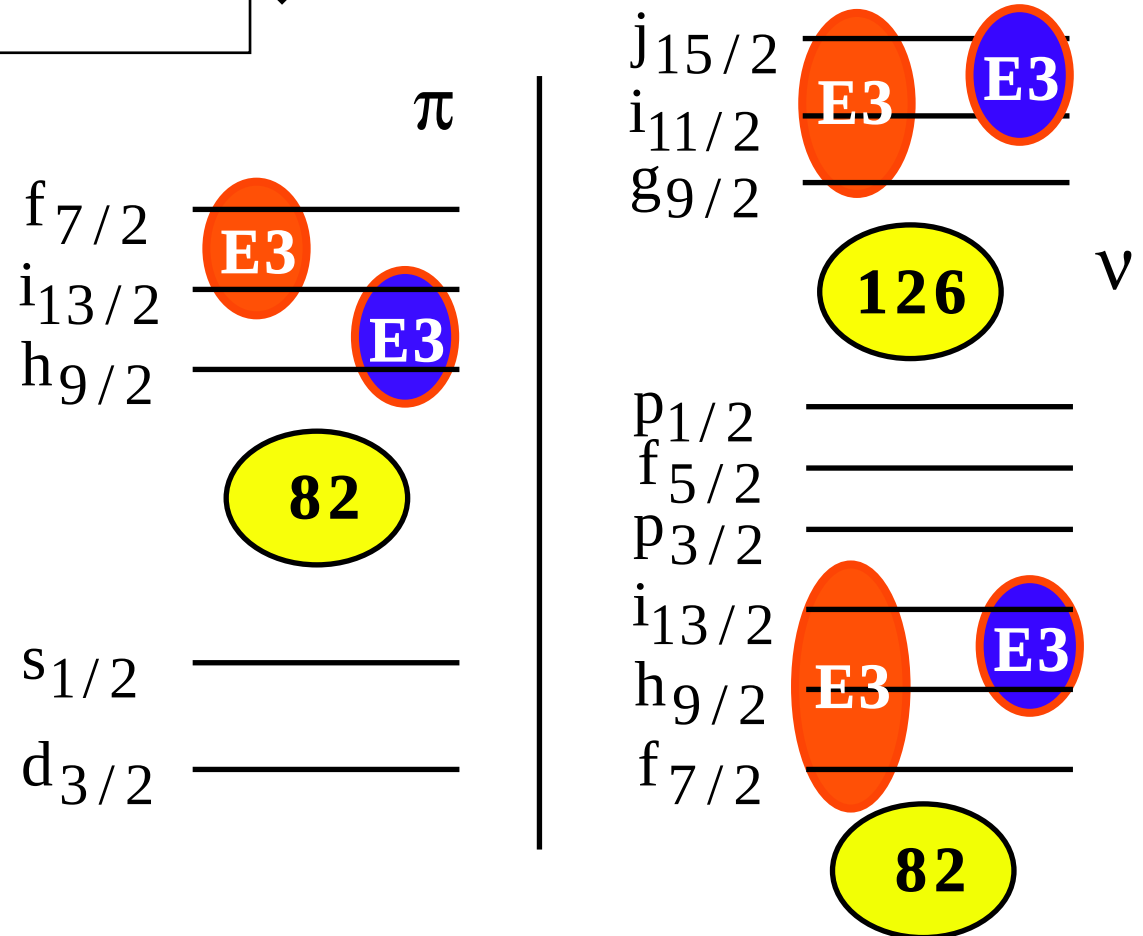
← collective E3 admixtures

"type-A" E3 ~ 22 W.u.
(stretched)

or $\pi i_{13/2} \rightarrow f_{7/2}$
or $\nu j_{15/2} \rightarrow g_{9/2}$

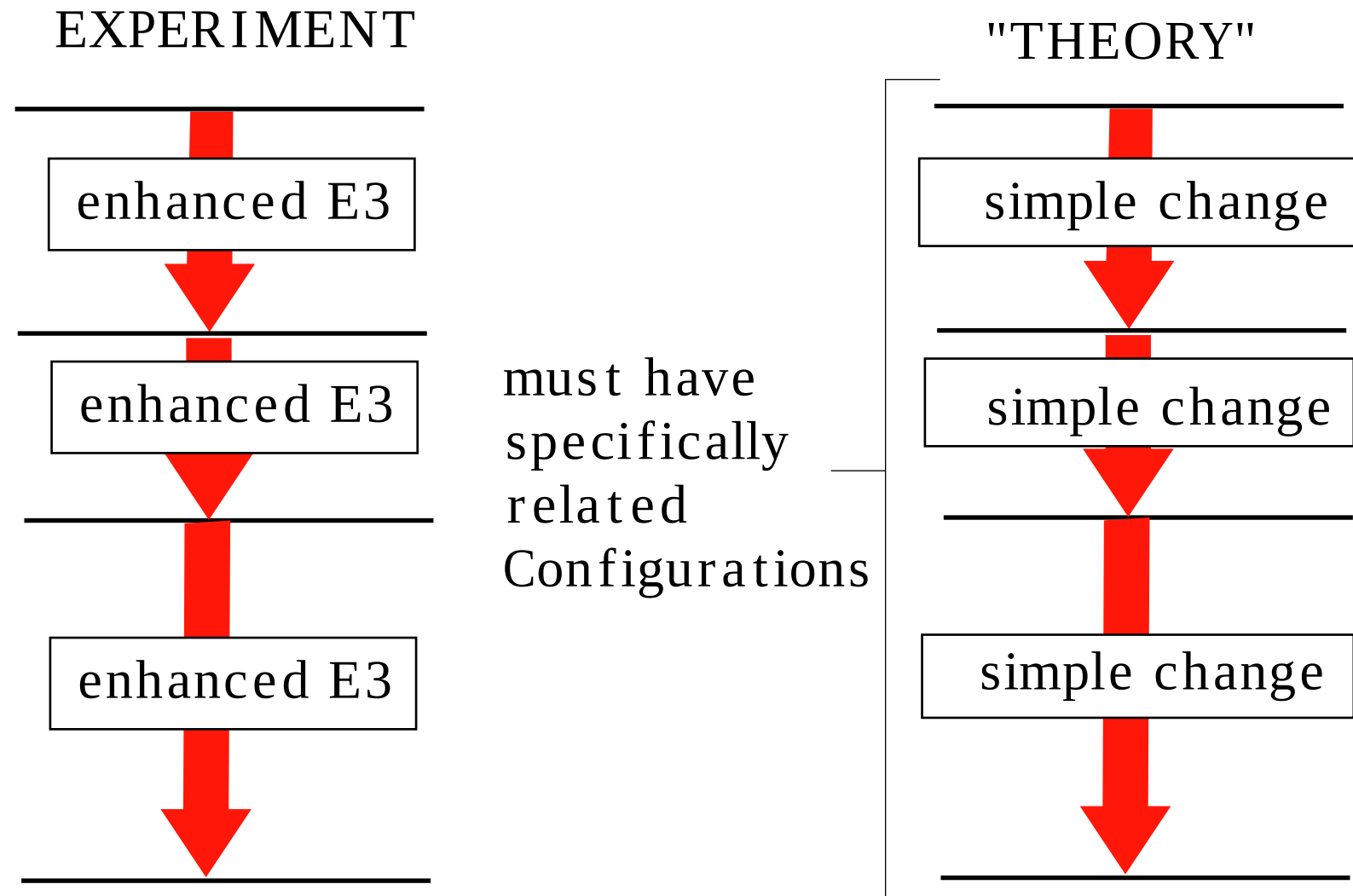
"type-B" E3 ~ 3 W.u.
(spin-flip)

or $\pi i_{13/2} \rightarrow h_{9/2}$
or $\nu j_{15/2} \rightarrow i_{11/2}$



see Bergstrom and Fant, Phys. Scr. 31 (1985) 26 and many others

A simple proposition...



**Low and High-Spin States (Isomers) in ^{212}Rn
Shell Model View
[Extremely Brief]**

Semi-Empirical Shell-Model (ESM):

- Single-Particle Energies + weighted sum of two-body interaction matrix elements

The shell-model Hamiltonian for a many-body system may be written as

$$H = \sum_i H_i + \sum_{i>j} H_{i,j}, \quad (1)$$

where H_i and $H_{i,j}$ are single-particle and two-body terms, respectively.

In terms of the excitation energy of a state Ψ , we can write:

$$E_{\text{ex}}(\Psi) = \varepsilon_0 + \sum \langle j_a, j_b \rangle_J, \quad (2)$$

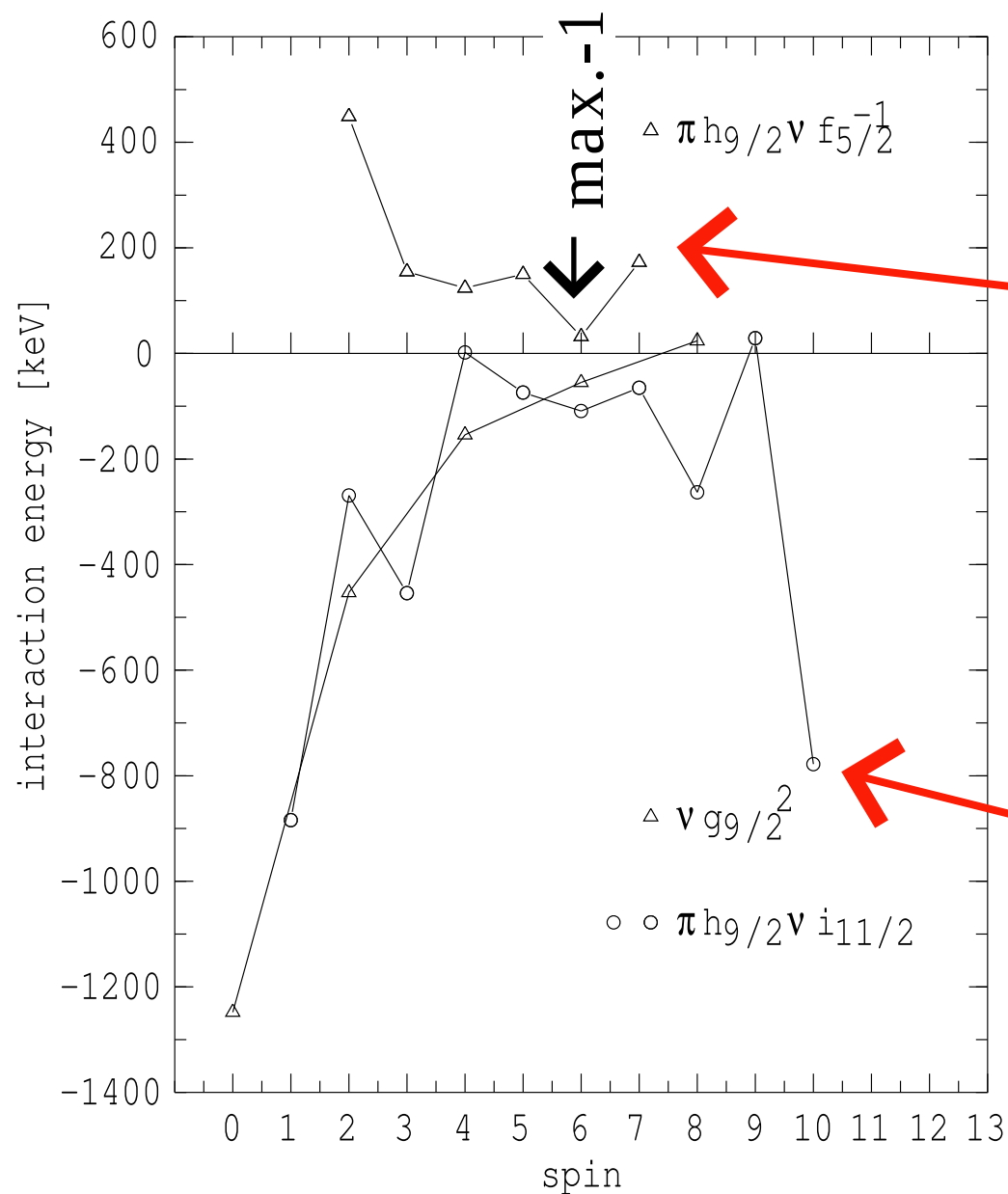
where ε_0 denotes the difference between the single-particle energy and the ground-state energy and $\langle j_a, j_b \rangle_J$ are the two-body residual interaction energies. In the present calculations interaction energies are taken from experiment where possible, and from theoretical models (Kuo and Herling [20]) where it is not. A full list of the interactions used in this work is tabulated in [21].

Blomqvist :

Proc Argonne Symposium, 1979 ANL/PHY-79-4, p155

See: Bayer et al, NPA 694 (2001) 3; NPA 650 (1999) 3; etc.

Typical residuals

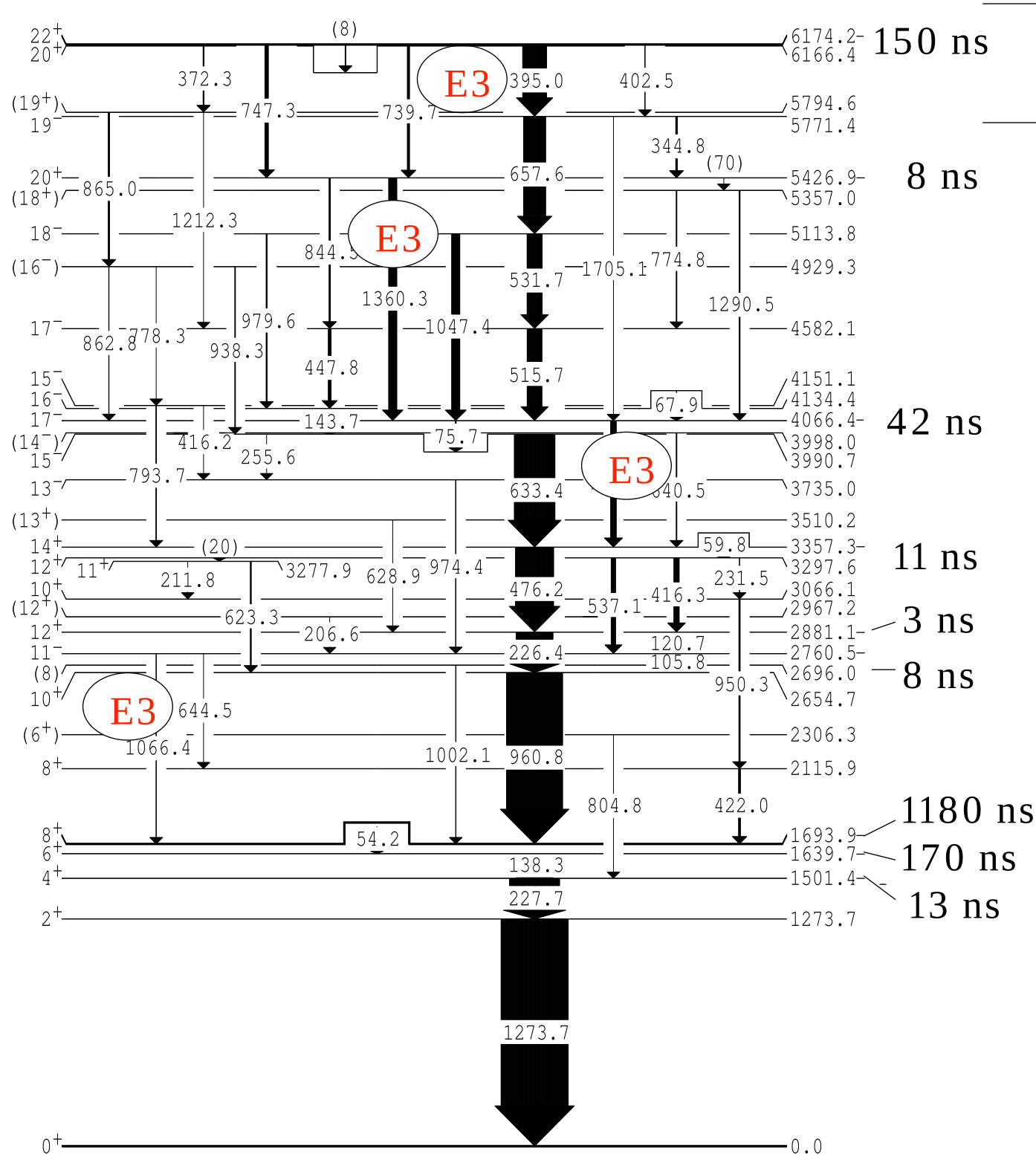


proton, neutron-hole

repulsive
for mutual alignment,
even for low-j neutron-hole

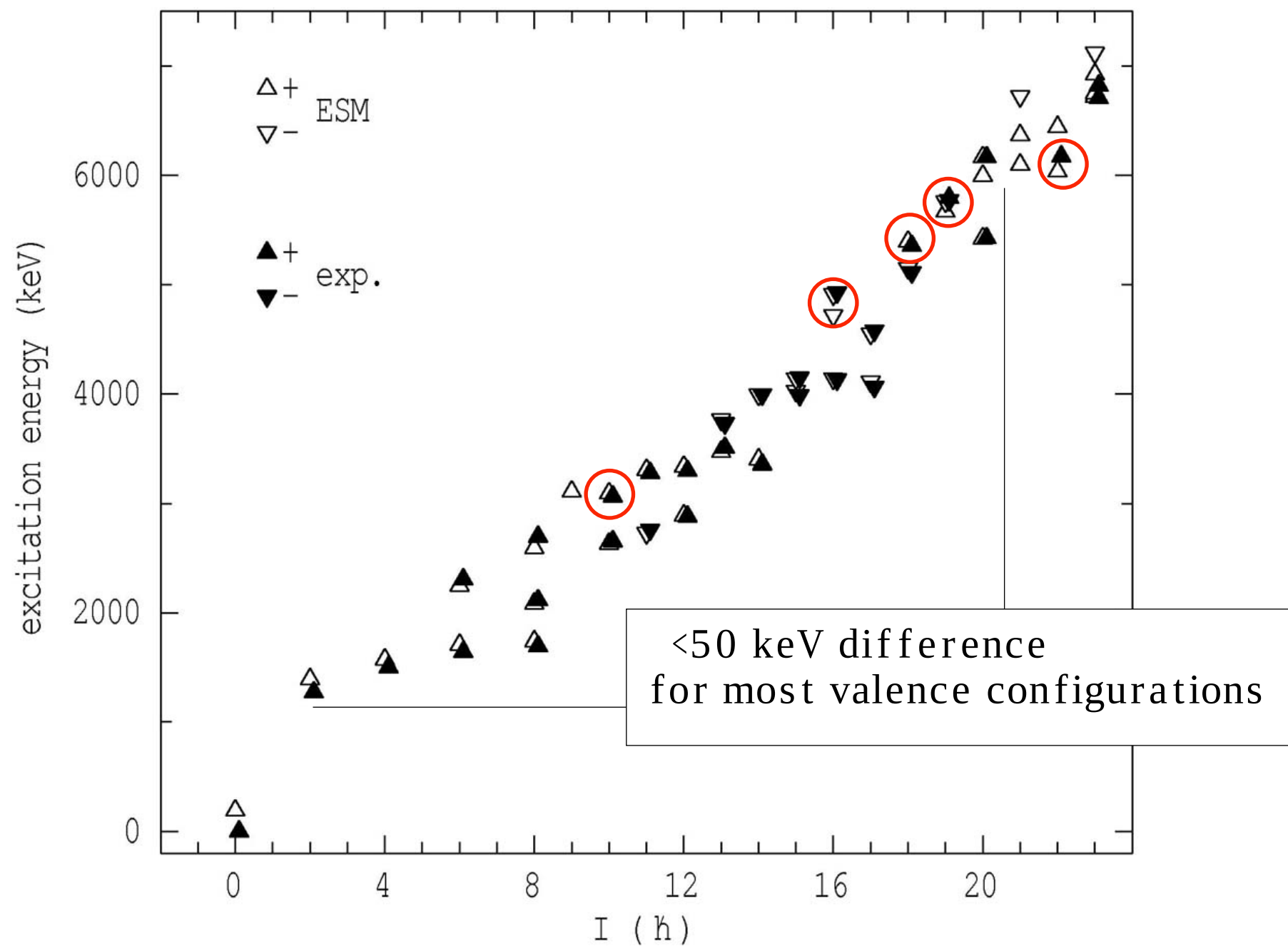
proton, neutron

strongly-attractive
for mutual alignment



22^+ , 20^+ and 19^-
core-excited states

^{212}Rn Z=86, N= 126 - Experiment and ESM - low



Strengths (sampler)

J^π		E(keV)	$\sigma\lambda$	W.u.	type
22 ⁺	20 ⁺ ₂	7.6	E2	4(1)	$\pi[h^3i] \nu p^{-1}g$ multiplet
	20 ⁺ ₁	747	E2	3.6(7)x10 ⁻⁵	core-excited to valence
	19 ⁻ ₁	403	E3	44(11)	core-excited to core-excited
					$\pi \tilde{i}_{13/2}$ to $\tilde{f}_{7/2}$ type-A
20 ⁺ ₁	18 ⁺	70	E2	2.7(5)	valence multiplet
	17 ⁻ ₂	845	E3	37(8)	valence to valence
					$\pi \tilde{i}_{13/2}$ to $\tilde{f}_{7/2}$
	17 ⁻ ₁	1360	E3	7(1)	valence to valence
					$\pi \tilde{i}_{13/2}$ to $\tilde{h}_{9/2}$ type-B

**Maximum - J ;
Single (I), Double (II), Triple(III) -
Neutron Core Excitations**

valence protons

$\pi [h^3f]_{17^-}$

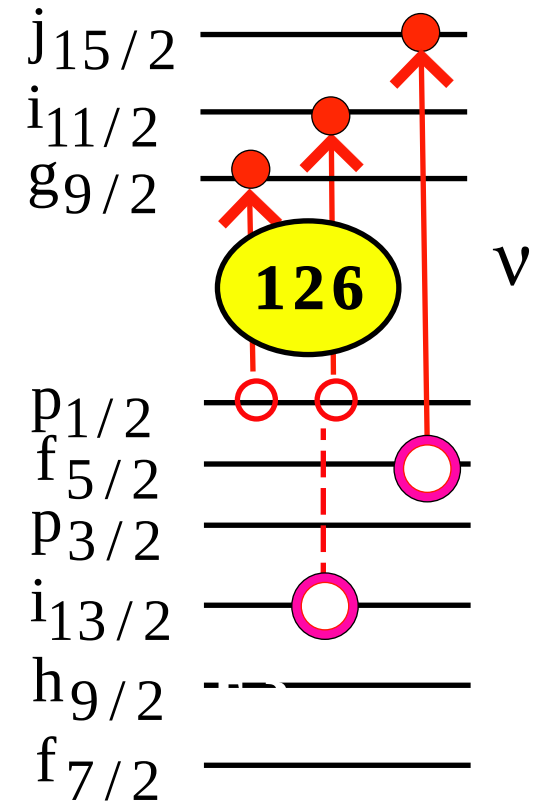
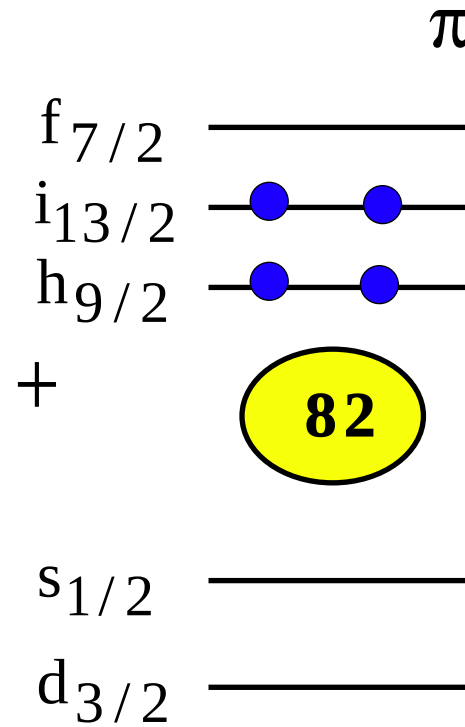
$\pi [h^2i^2]_{20^+}$

+ $\nu [p^{-1}g]_{5^-}$
 $\nu [p^{-1}i]_{6^-}$ **I**
 $\nu [p^{-1}j]_{8^+}$ **~28h**

$\nu [f^{-1}g]_{7^-}$ **~30h**

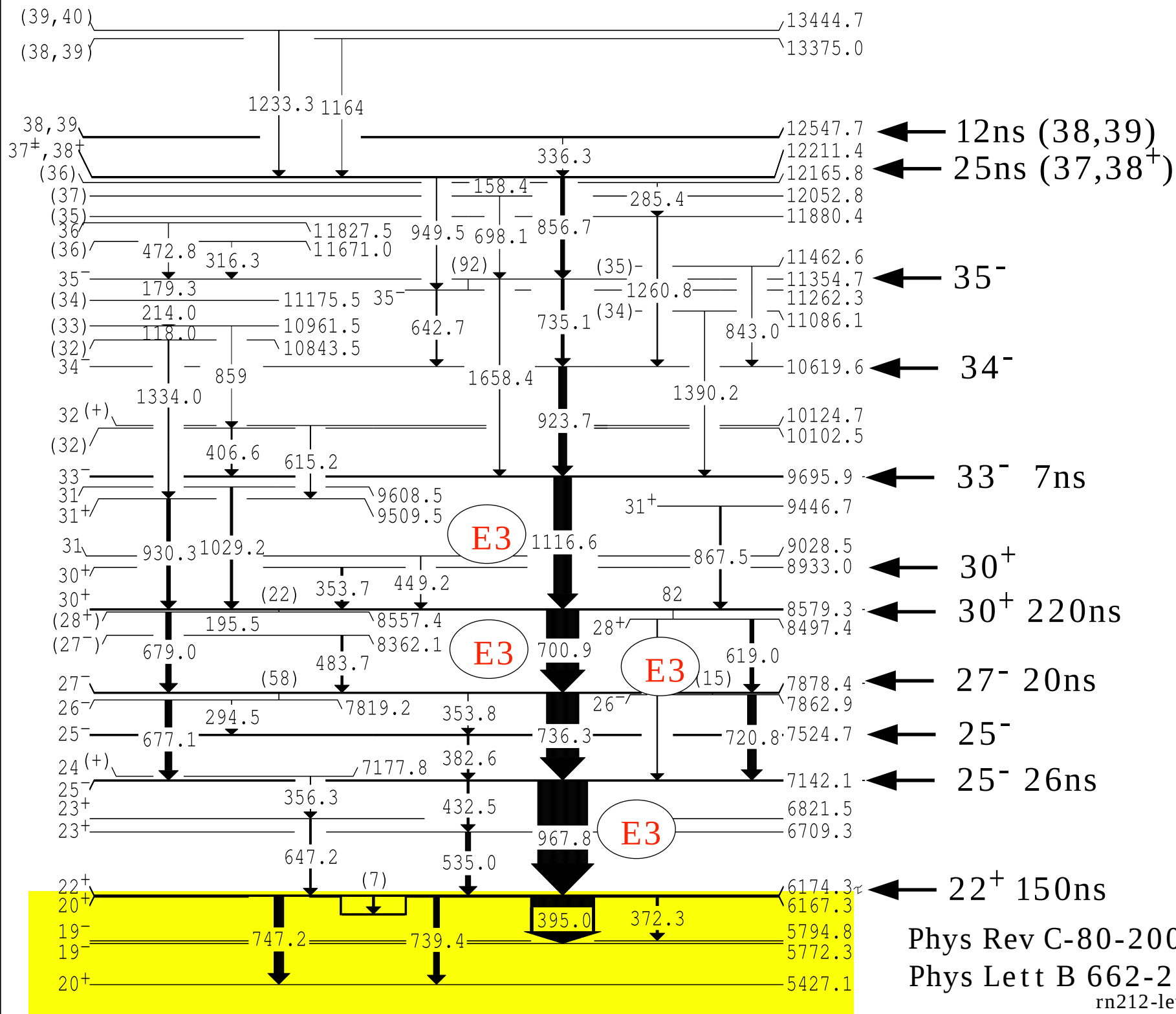
+ $\nu [p^{-2}gi]_{10^+}$ **II**
 $\nu [p^{-2}j^2]_{14^+}$ **~36h**
 $\nu [p^{-1}f^{-1}gj]_{16^-}$

$\nu [p^{-1}i^{-1}ij] \dots 20^+$ **~40h**



Mutual Alignment of high-j
neutron holes/proton-particles
- repulsive interaction

+ $\nu [p^{-2}f^{-1}gij] \dots 20^+$ **III**
~40h

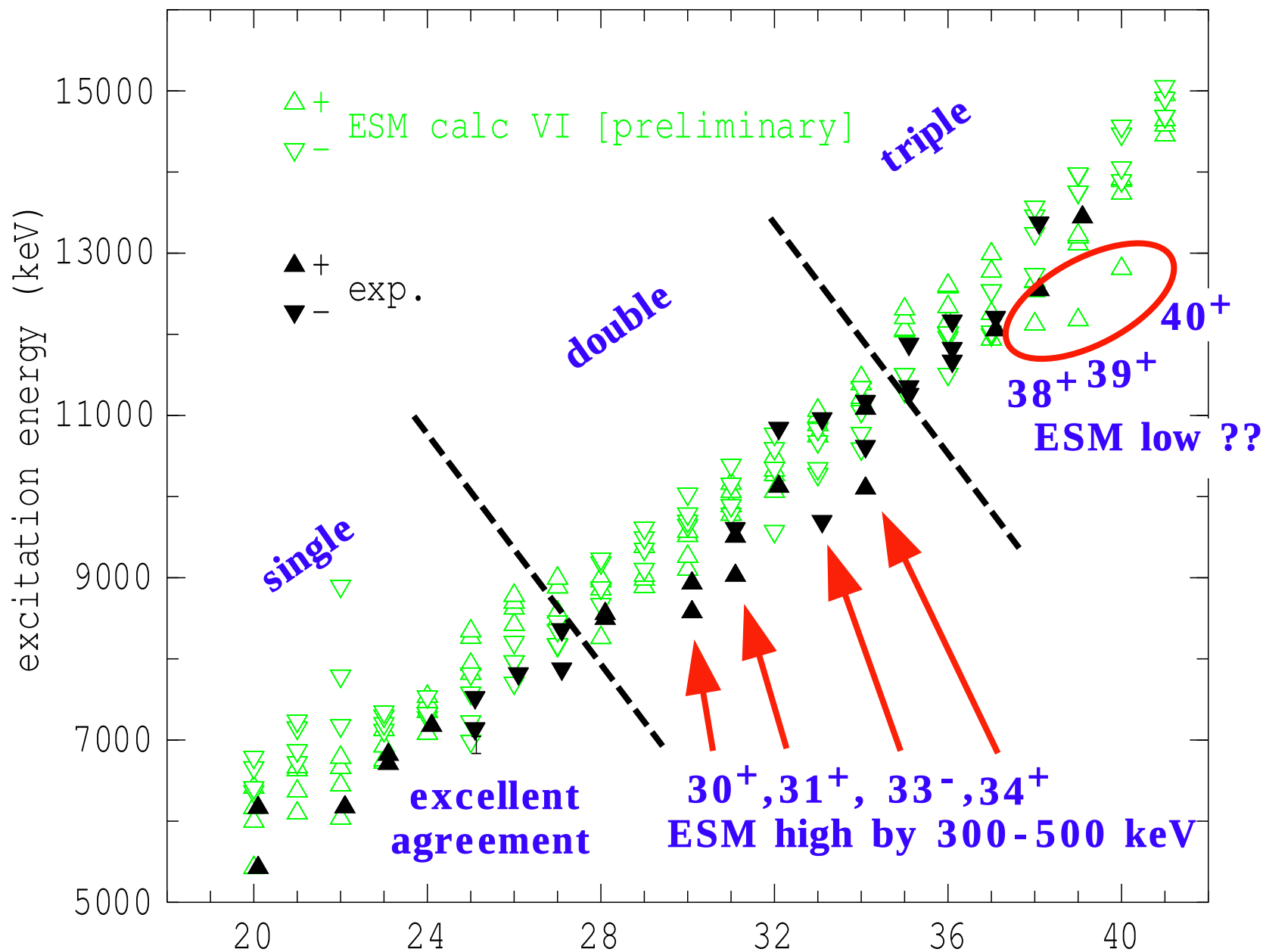


Phys Rev C-80-2009-05430

Phys Lett B 662-2008-19

rn212-levels-surrey-I.md

^{212}Rn Z=86, N= 126 - Experiment and ESM - preliminary



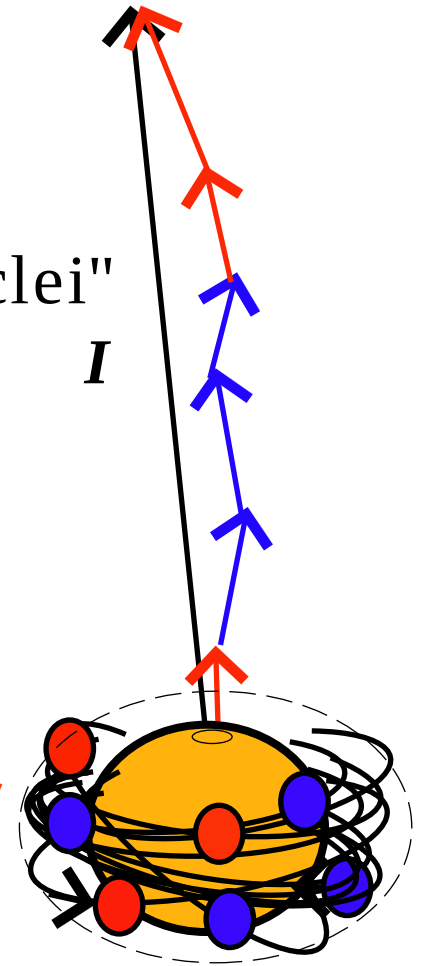
Deformed View (DIPM)

Induced Deformation

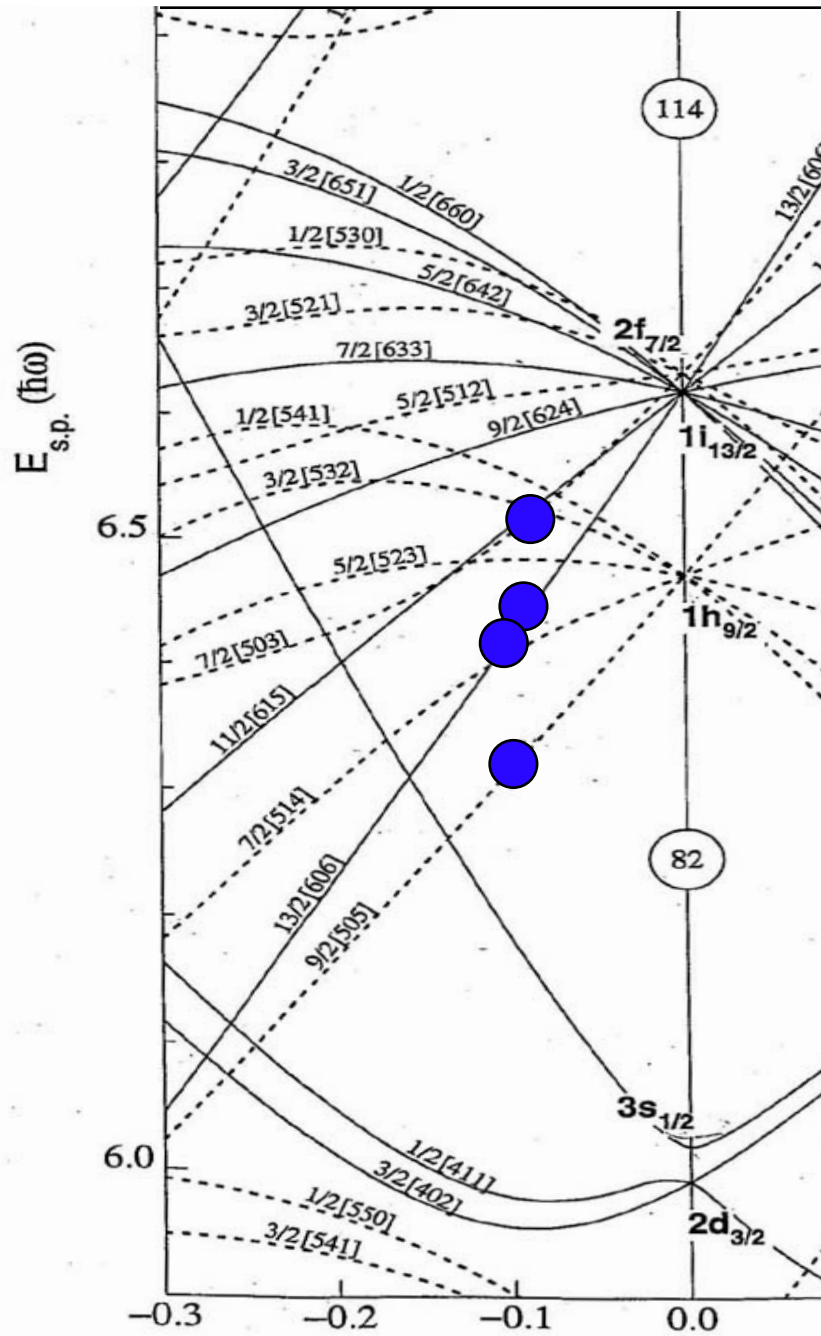
"New Era of High-Spin Studies in Trans-lead Nuclei"

Deformed Independent Particle Model (DIPM)
Matsuyanagi et al Nucl Phys A 307 (1978) 253

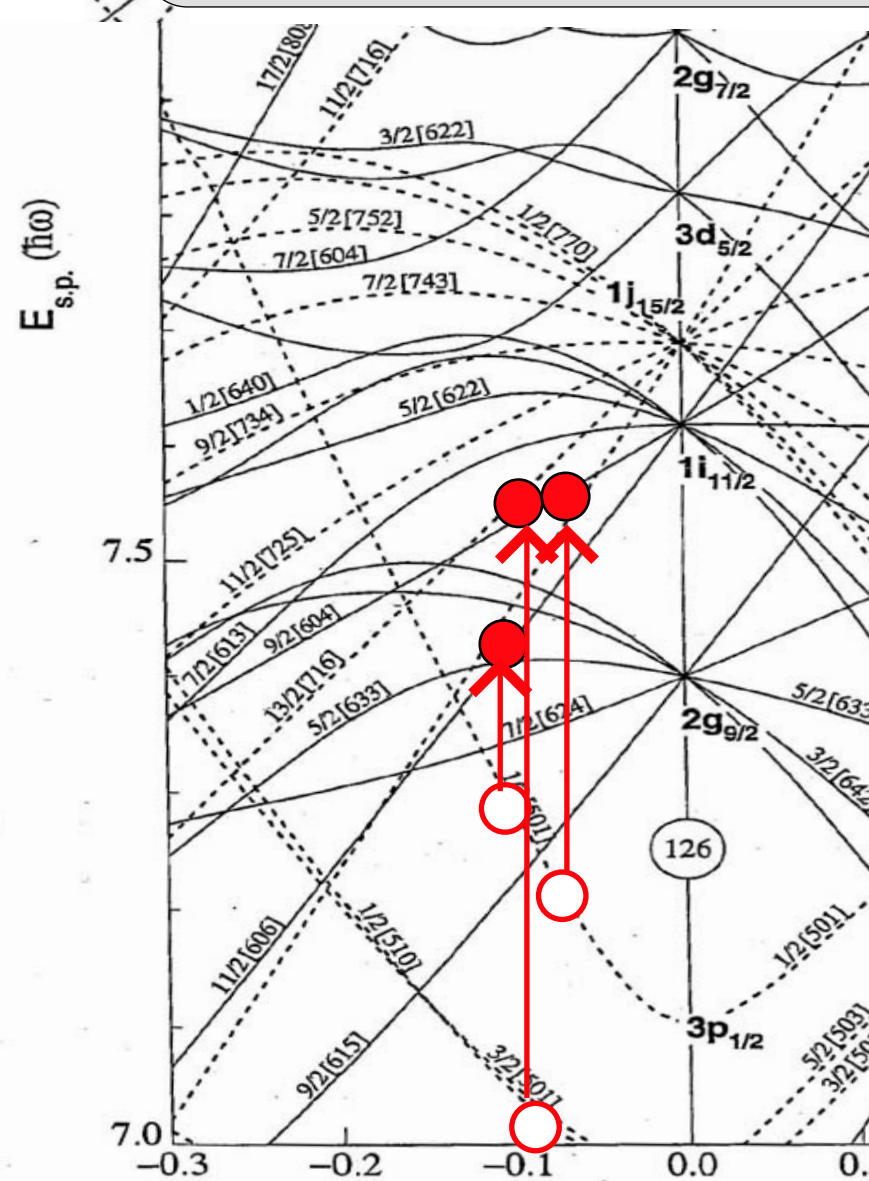
aligned high-spin states
equatorial nucleons congregate
- oblate deformation ?



Nilsson...

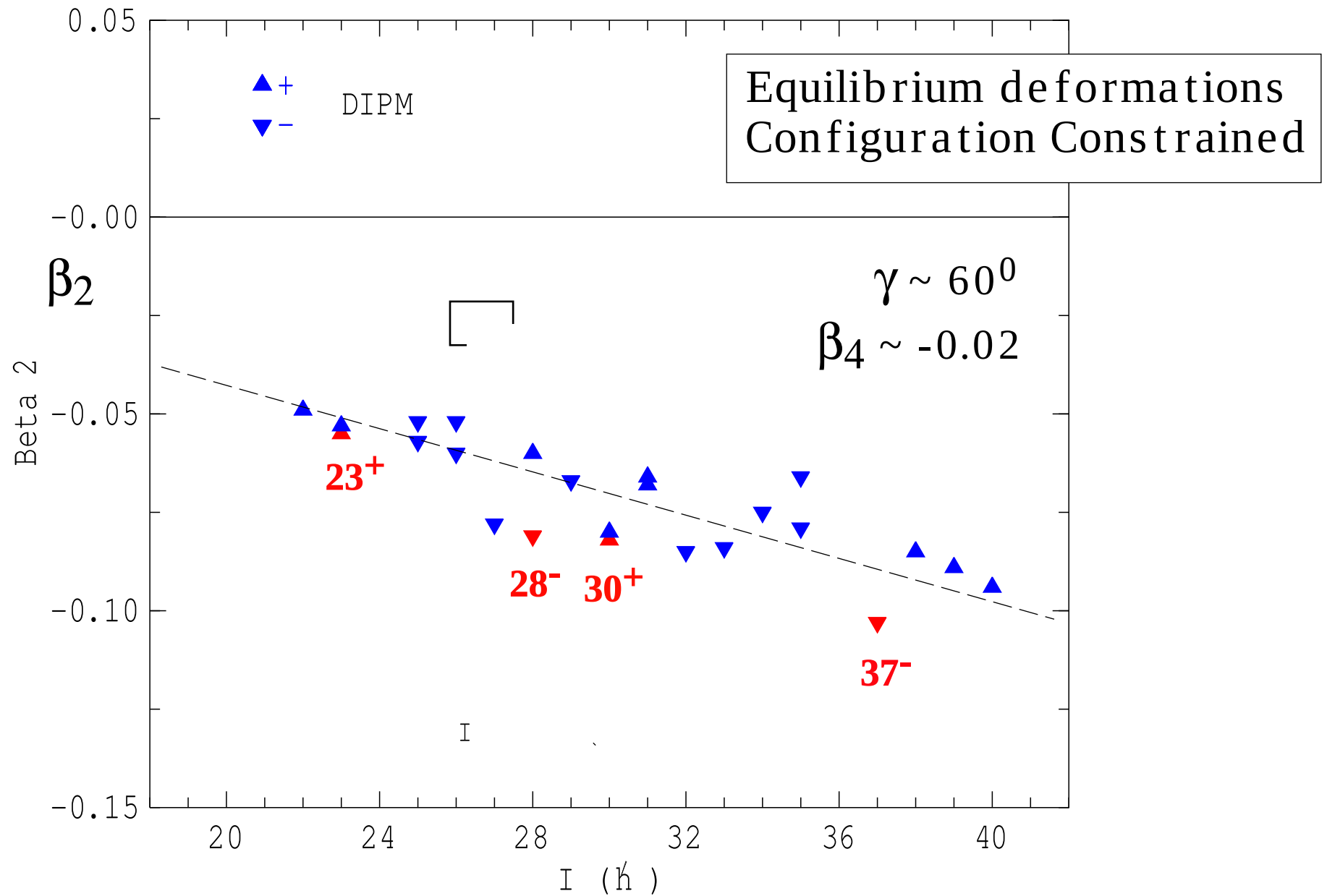


Protons.

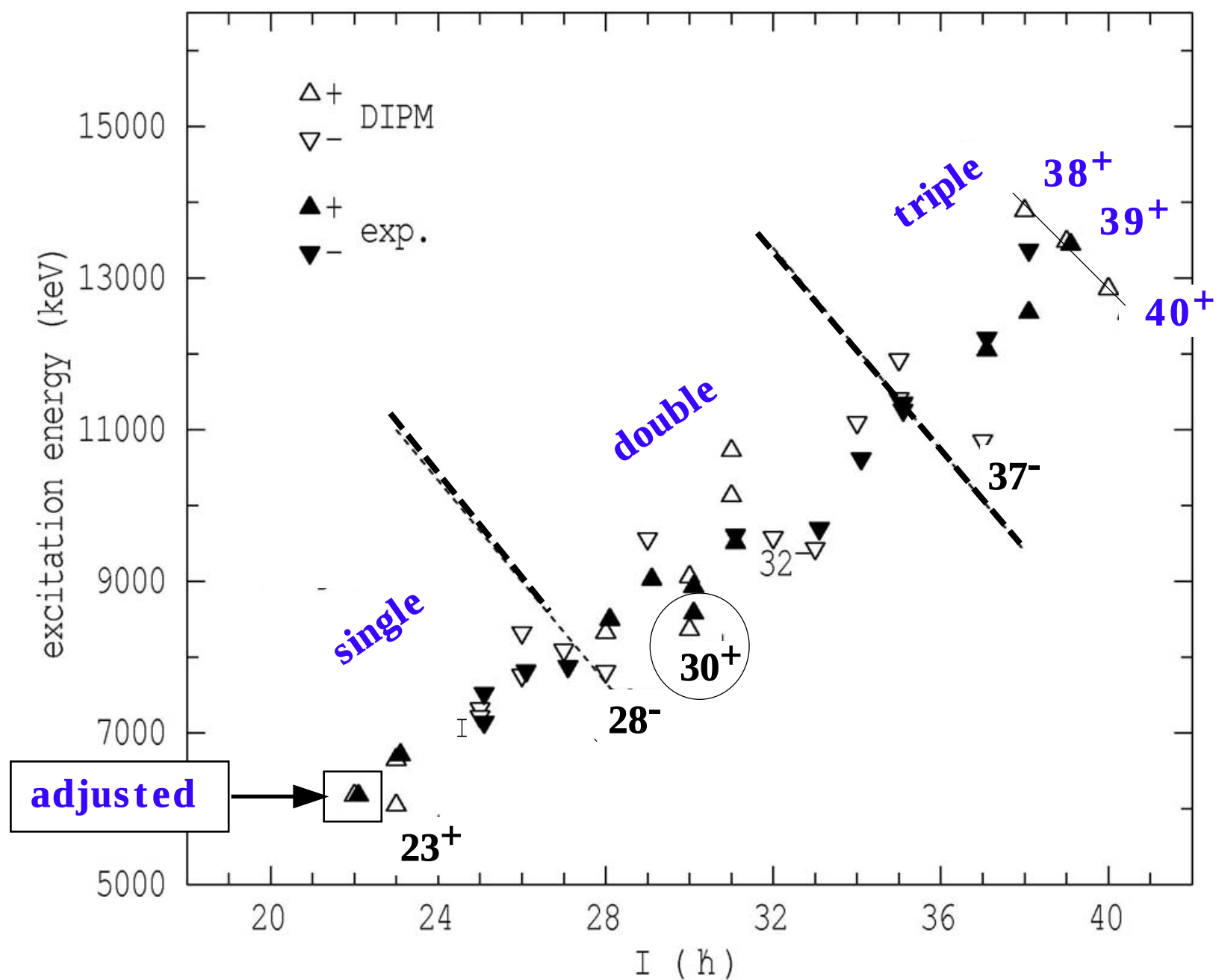


Neutrons.

DIPM - predicted deformations (Xu,Liu)



^{212}Rn Z=86, N= 126 - Experiment and DIPM



**DIPM Configuration assignments
do not match E3 transitions**

DIPM Assignment - Matsuyanagi et al., Andersson et al; deVoight review...

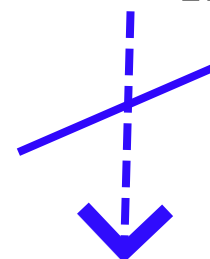
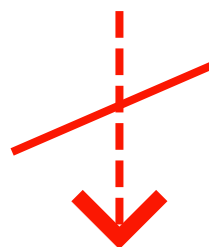
**Strong E3 not
Possible**

30^+ ————— 220ns

701 31(3)

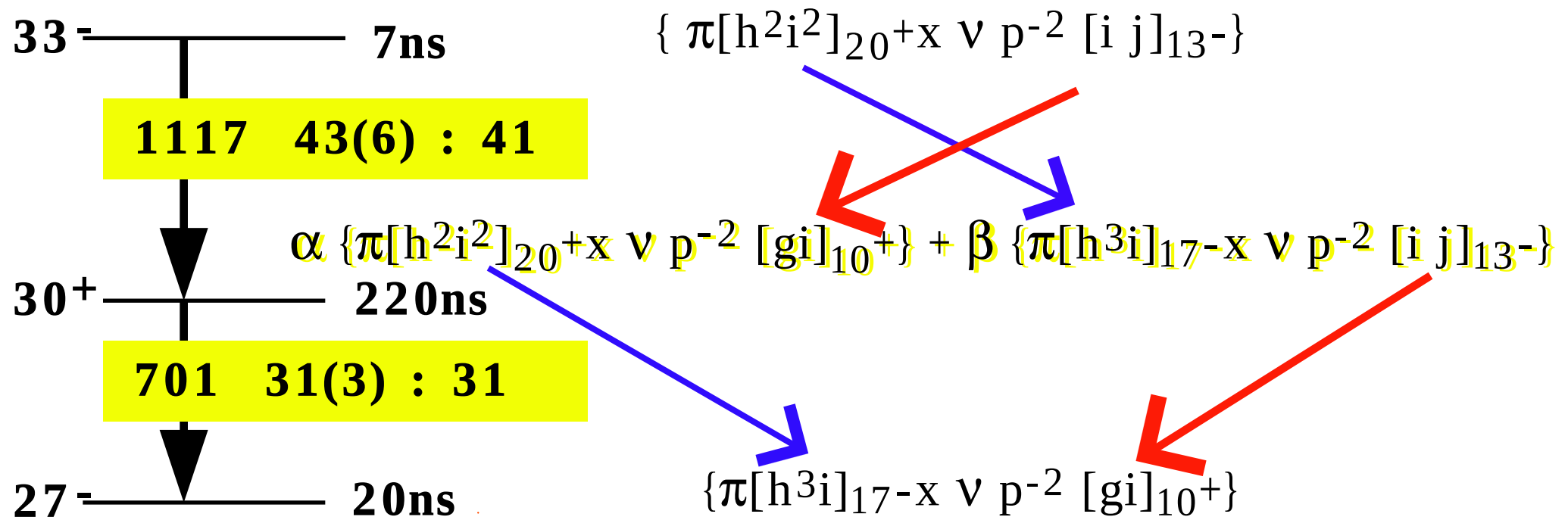
27^- ————— 20ns

$\pi[h^2fi]_{18-x} \vee p^{-2} [gj]_{12-}$



$\{\pi[h^3i]_{17-x} \vee p^{-2} [gi]_{10+}\}$

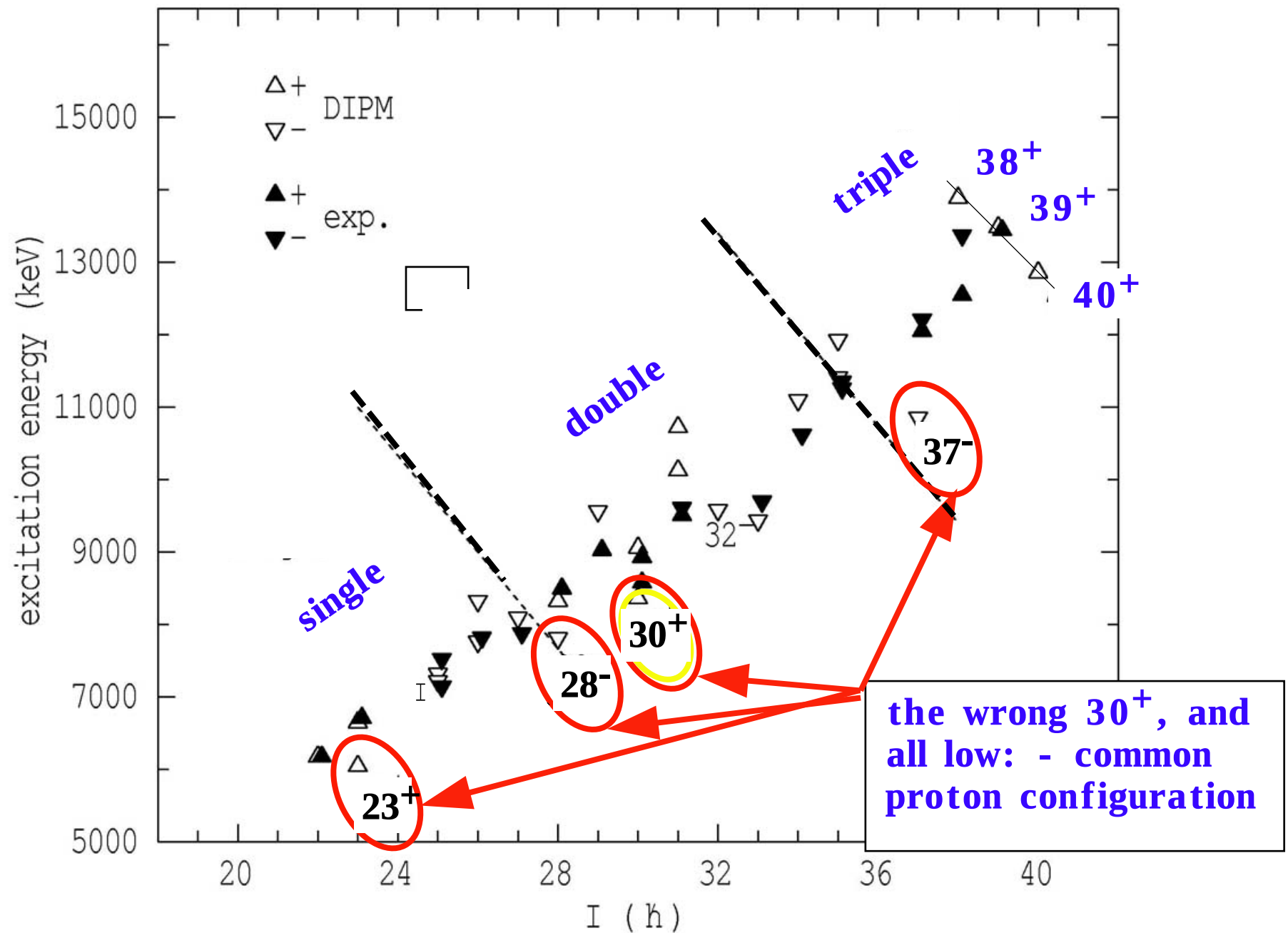
Multiple - Particle - Octupole - Coupling: MPOC



Poletti et al NPA 448 (1986) 189
 Dracoulis et al PLB 246 (1990) 31

**"Automatic" mixing
 Constructive Interference**

^{212}Rn Z=86, N= 126 - Experiment and DIPM - preliminary



Why is 18^- too low in DIPM?

$$23^+ \quad \pi (h^2 fi)_{18^-} \times V(p^{-1}g)_5^-$$

$$28^- \quad \pi (h^2 fi)_{18^-} \times V(p^{-2}gi)_{10}^+$$

$$30^+ \quad \pi (h^2 fi)_{18^-} \times V(p^{-2}gi)_{10}^+ \leftarrow \text{the original problem}$$

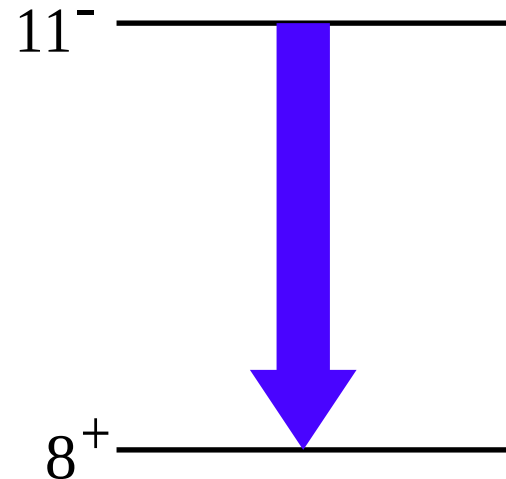
$$37^- \quad \pi (h^2 fi)_{18^-} \times V(p^{-2}f^{-1}gij)_{19}^+$$

CASE III; Pb-Po, 11⁻

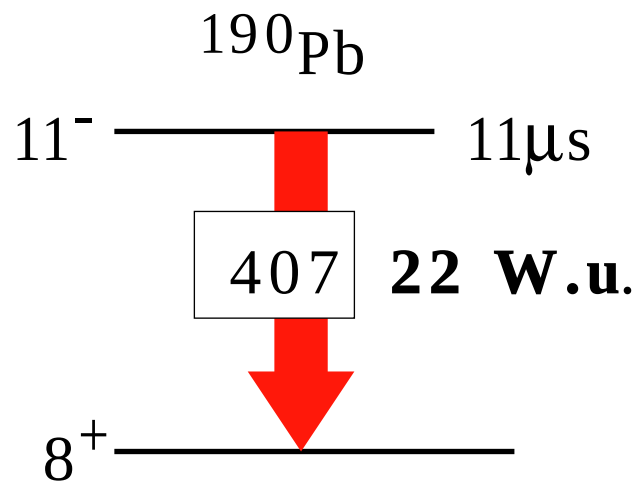
11^- to 8^+ E3 ; naive expectation

$\pi [i_{13/2} h_{9/2}] 11^-$ to $[h_{9/2}^2] 8^+$

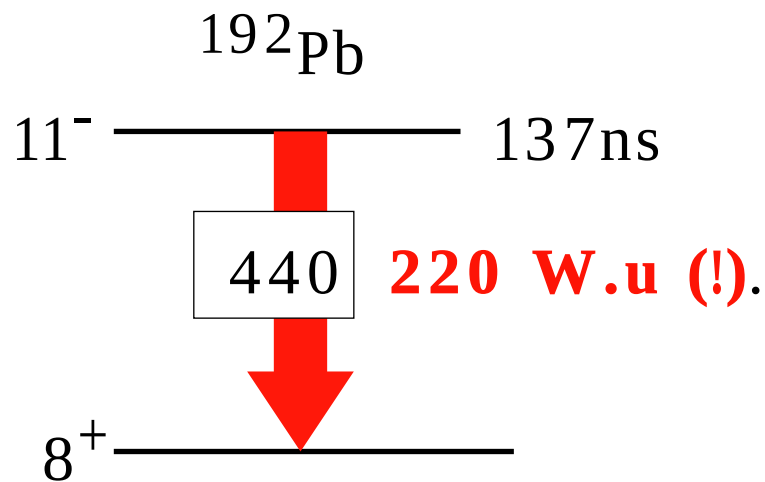
"type-B" E3 ~ 3 W.u.
(spin-flip)



11^- to 8^+ E3 ; emerging reality



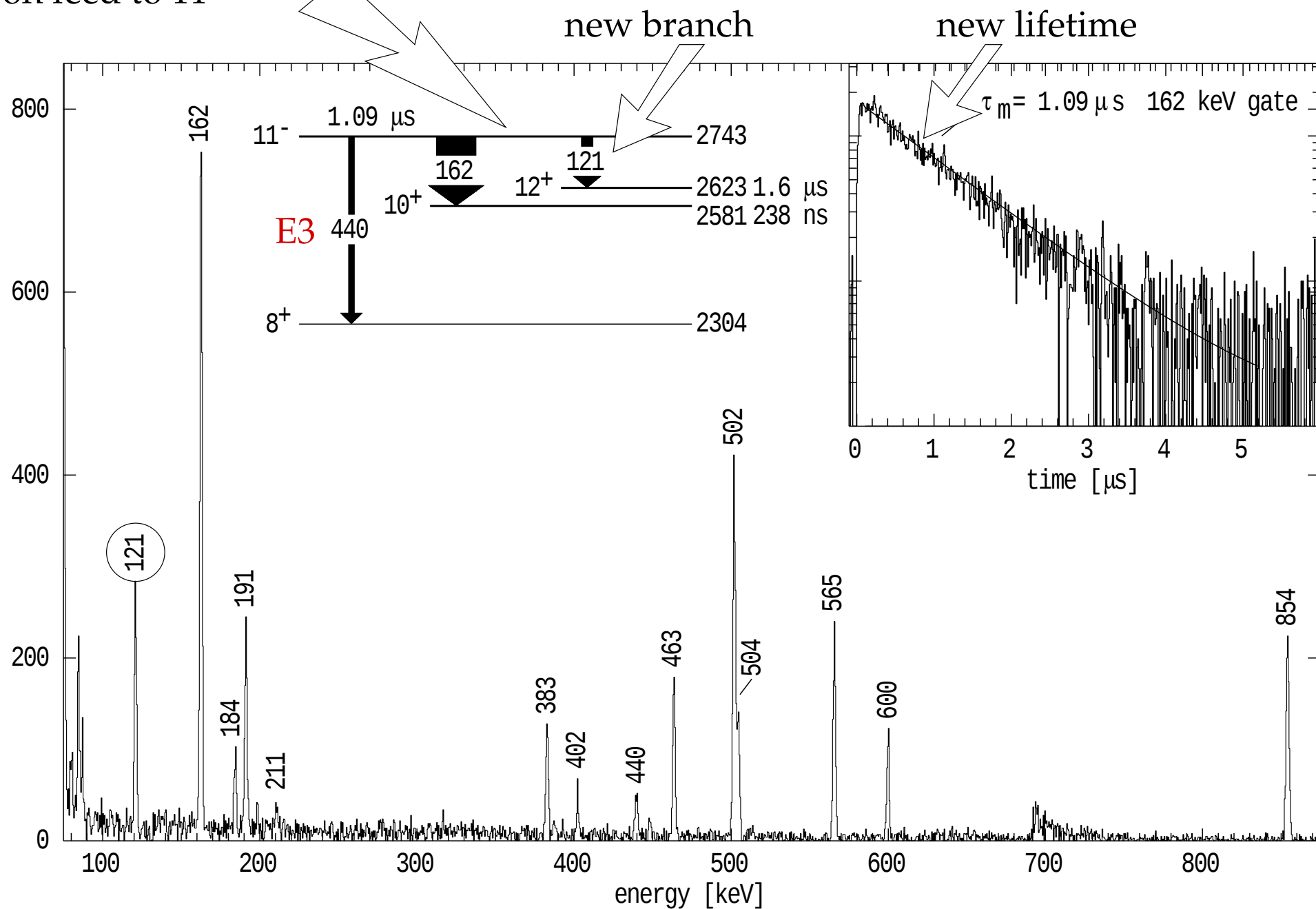
Dracoulis et al
PLB 432 (1998)37



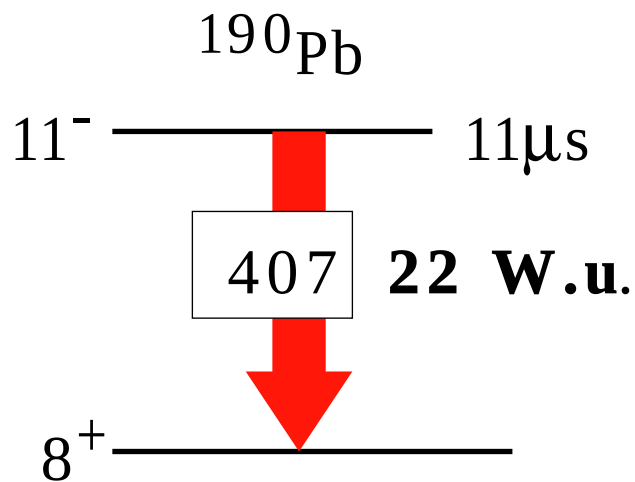
Literature
values of Lifetime and
Branching

delayed lines; gated
on feed to 11-

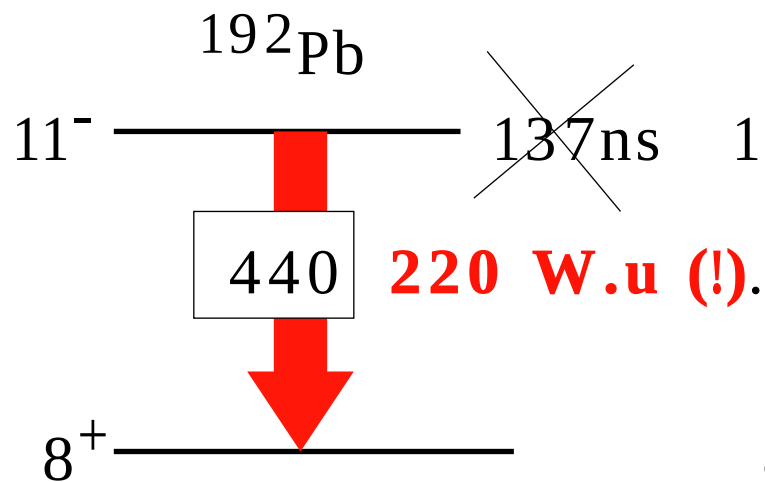
Pb-192 11- state



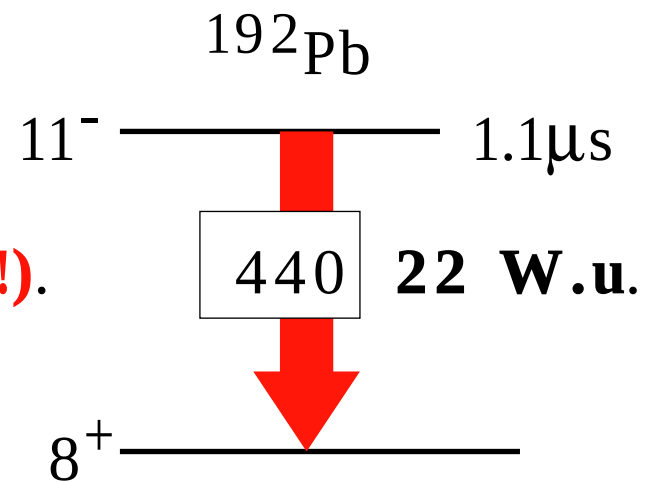
11^- to 8^+ E3 ; emerging reality



Dracoulis et al
PLB 432 (1998)37



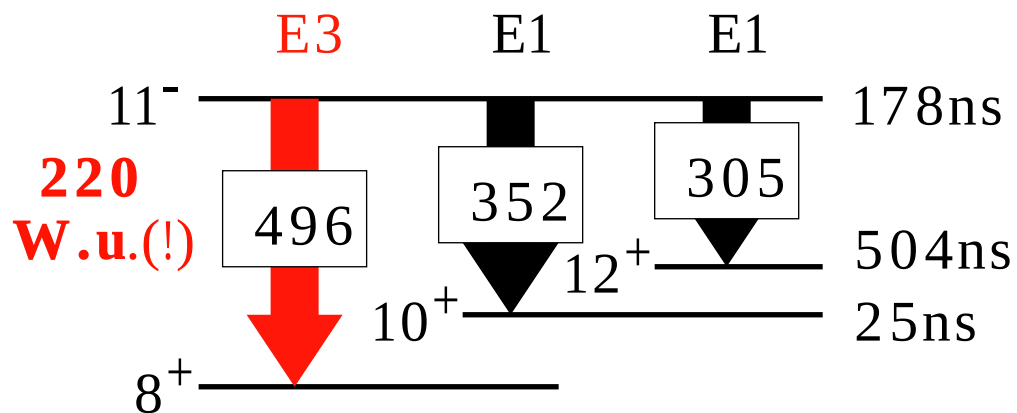
Literature
values of ~~Lifetime~~ and
~~Branching~~



Dracoulis et al
PRC 63 (2001)
061302

"type-A" E3 $\sim 20\text{ W.u}$

11^- to 8^+ E3 ; ^{194}Pb

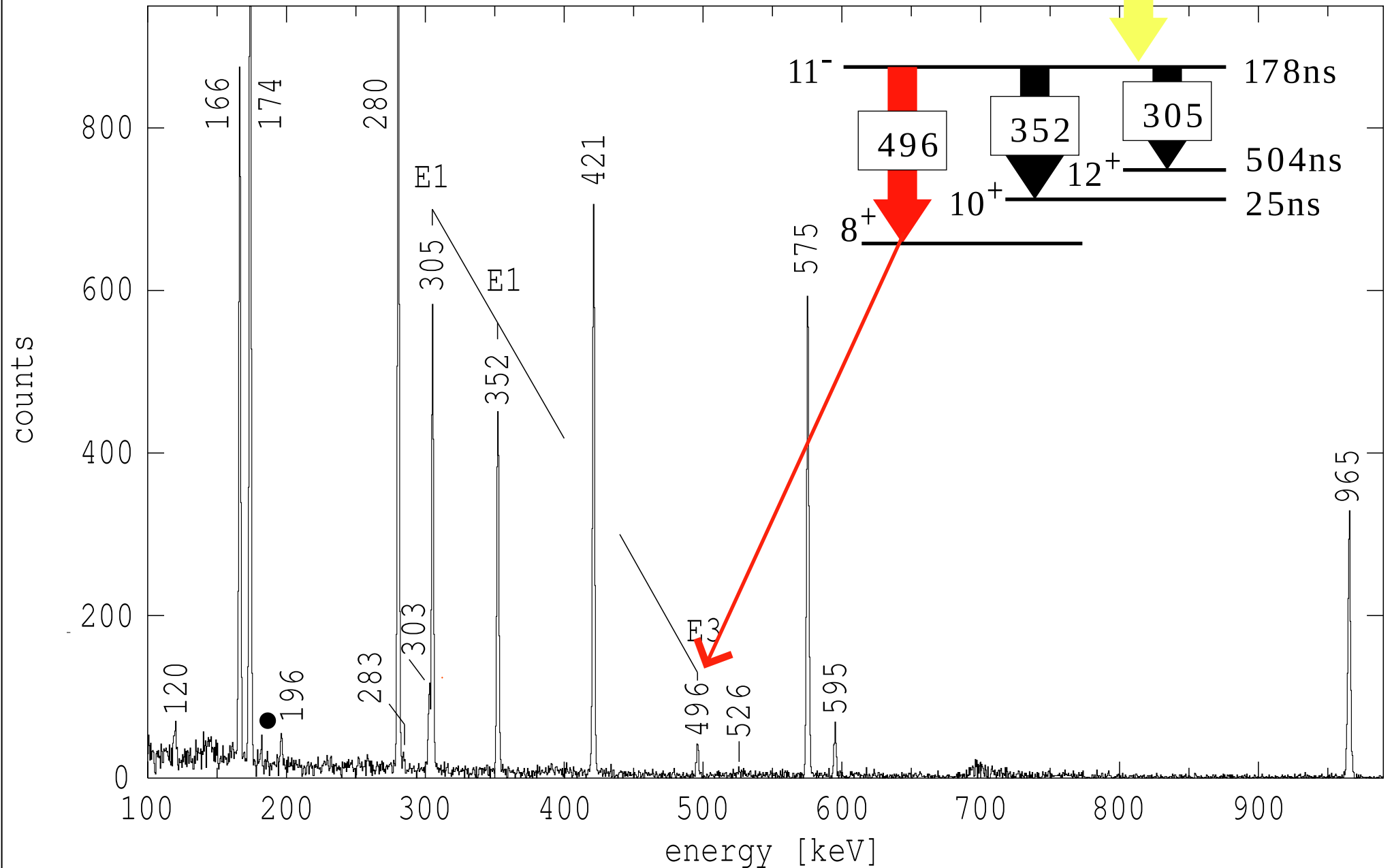


$$I_{\gamma}(496) = I_{\gamma}(352) = I_{\gamma}(305) ?$$

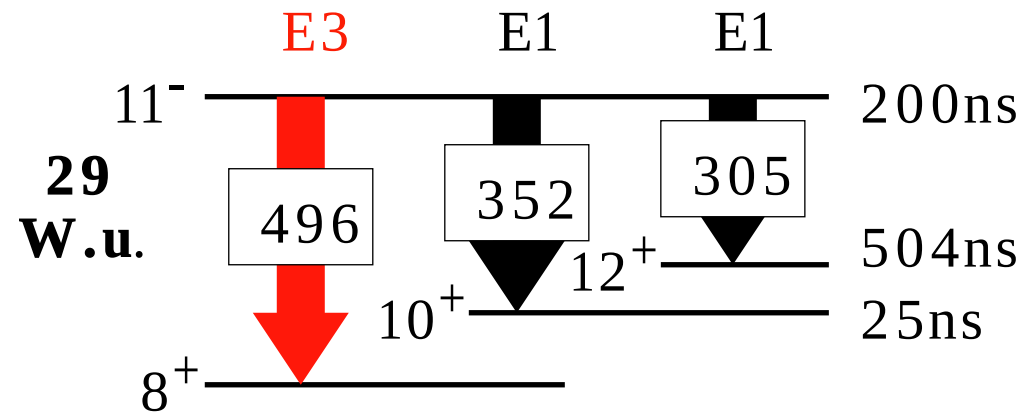
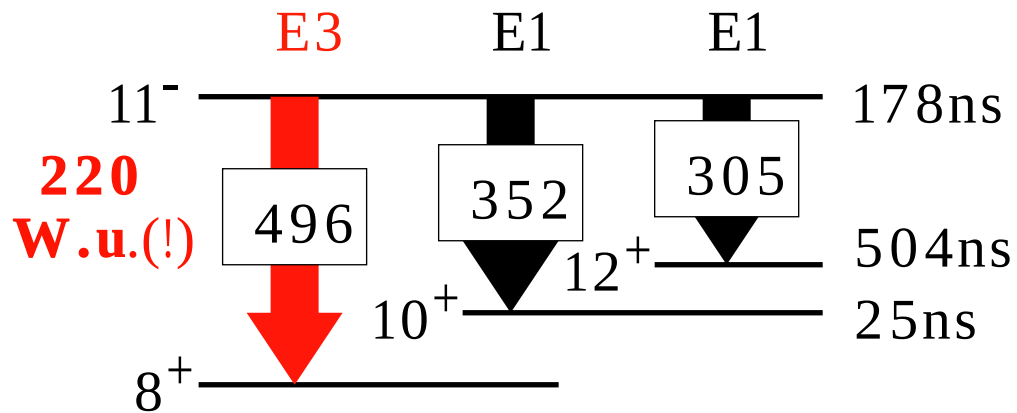
Van Ruyven et al (1986)
 Fant et al (1991)
 Kaci et al (2002)

11^- Isomer decay ^{194}Pb

γ -ray gates above
plus time gate



11^- to 8^+ E3 ; ^{194}Pb



$$I_{\gamma}(496) = I_{\gamma}(352) = I_{\gamma}(305) ?$$

$$I_{\gamma}(496) \ll I_{\gamma}(352) = I_{\gamma}(305) ?$$

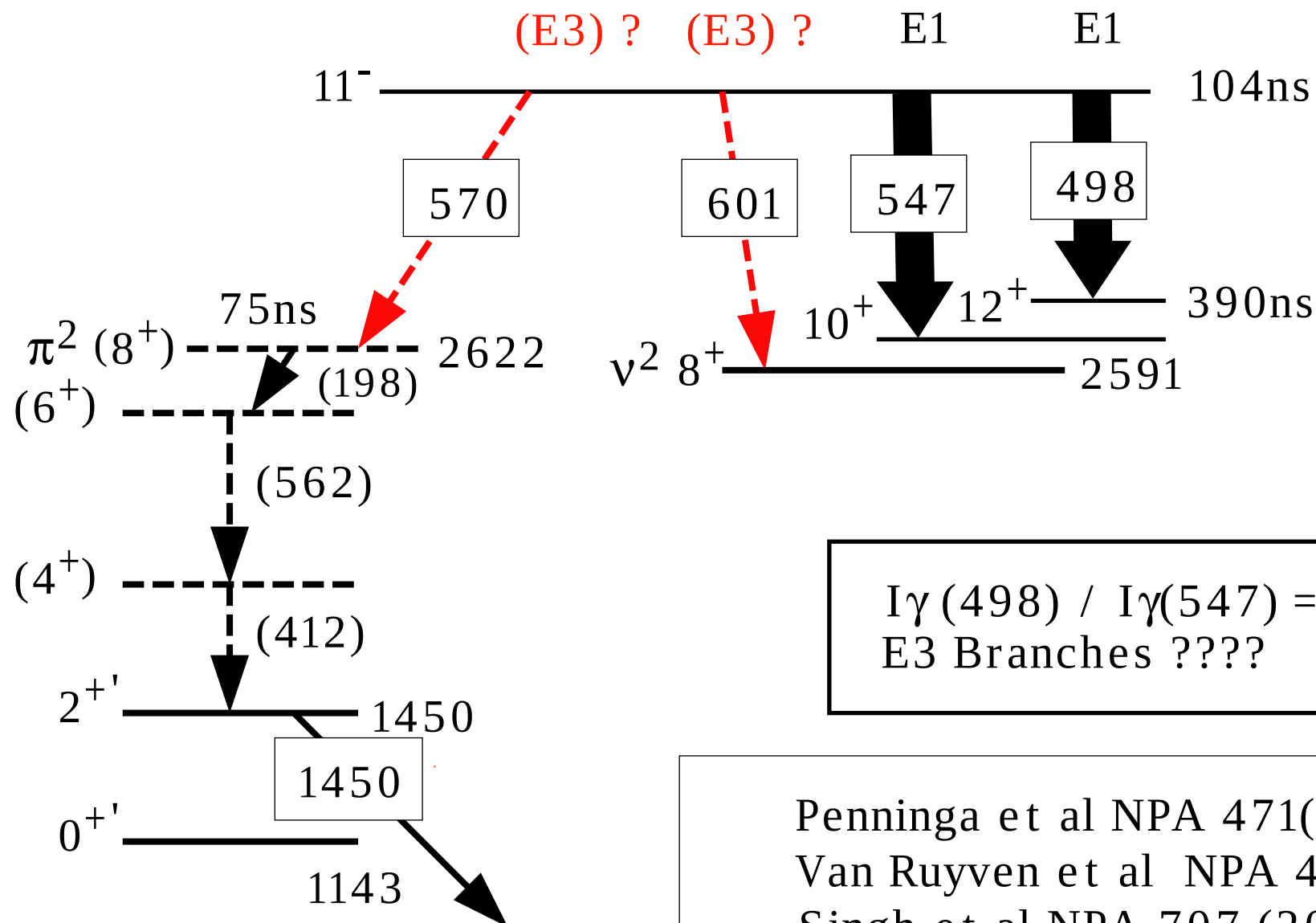
Van Ruyven et al (1986)
Fant et al (1991)
Kaci et al (2002)

"type-A" E3 $\sim 20 \text{ W.u.}$

...to be published

11⁻ to 8⁺ E3 ; ¹⁹⁶Pb challenge

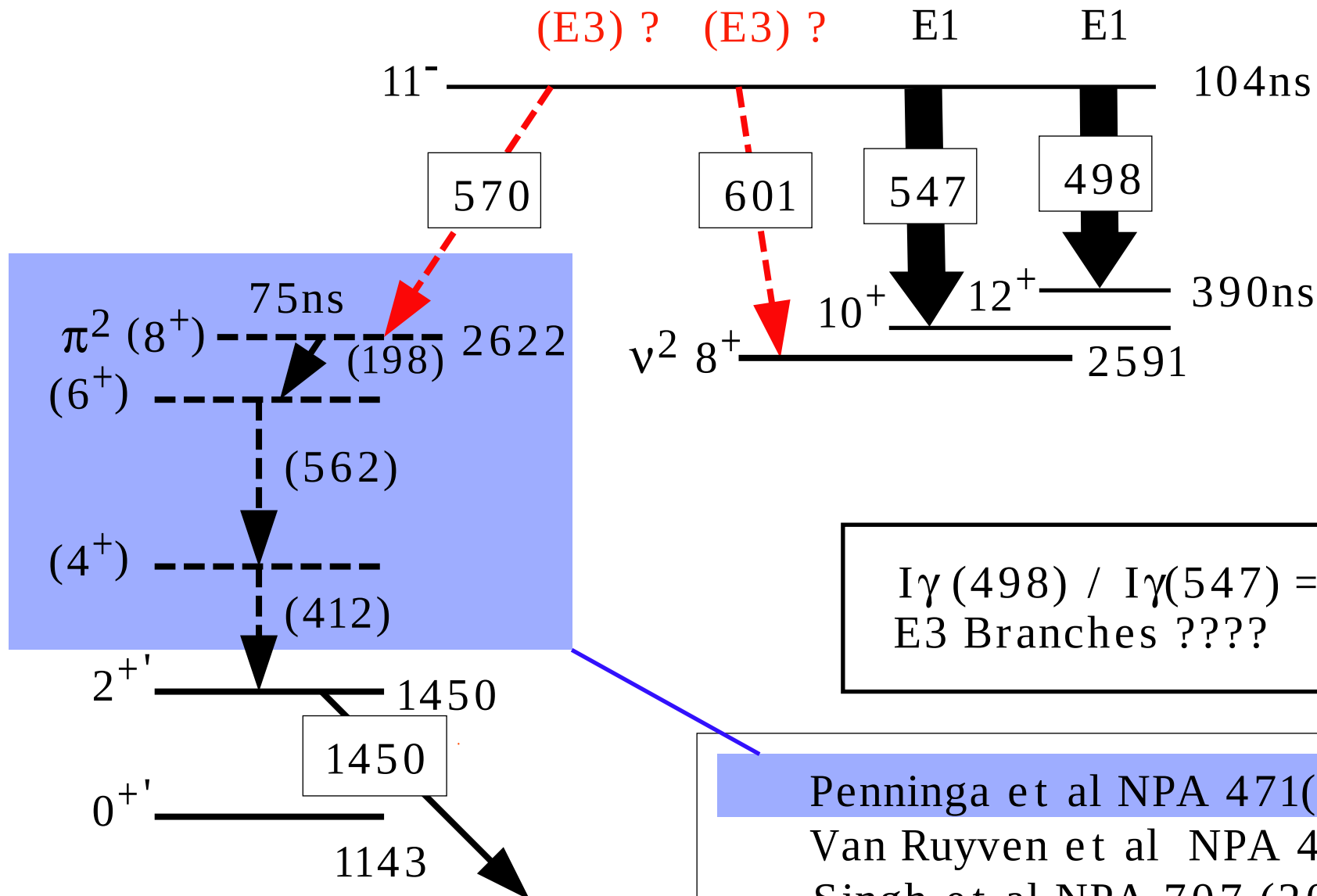
short- τ = weak branch



$I\gamma(498) / I\gamma(547) = \text{????}$
E3 Branches ????

Penninga et al NPA 471(1987)535
Van Ruyven et al NPA 449(1986)
Singh et al NPA 707 (2002) 3

11⁻ to 8⁺ E3 ; ¹⁹⁶Pb challenge

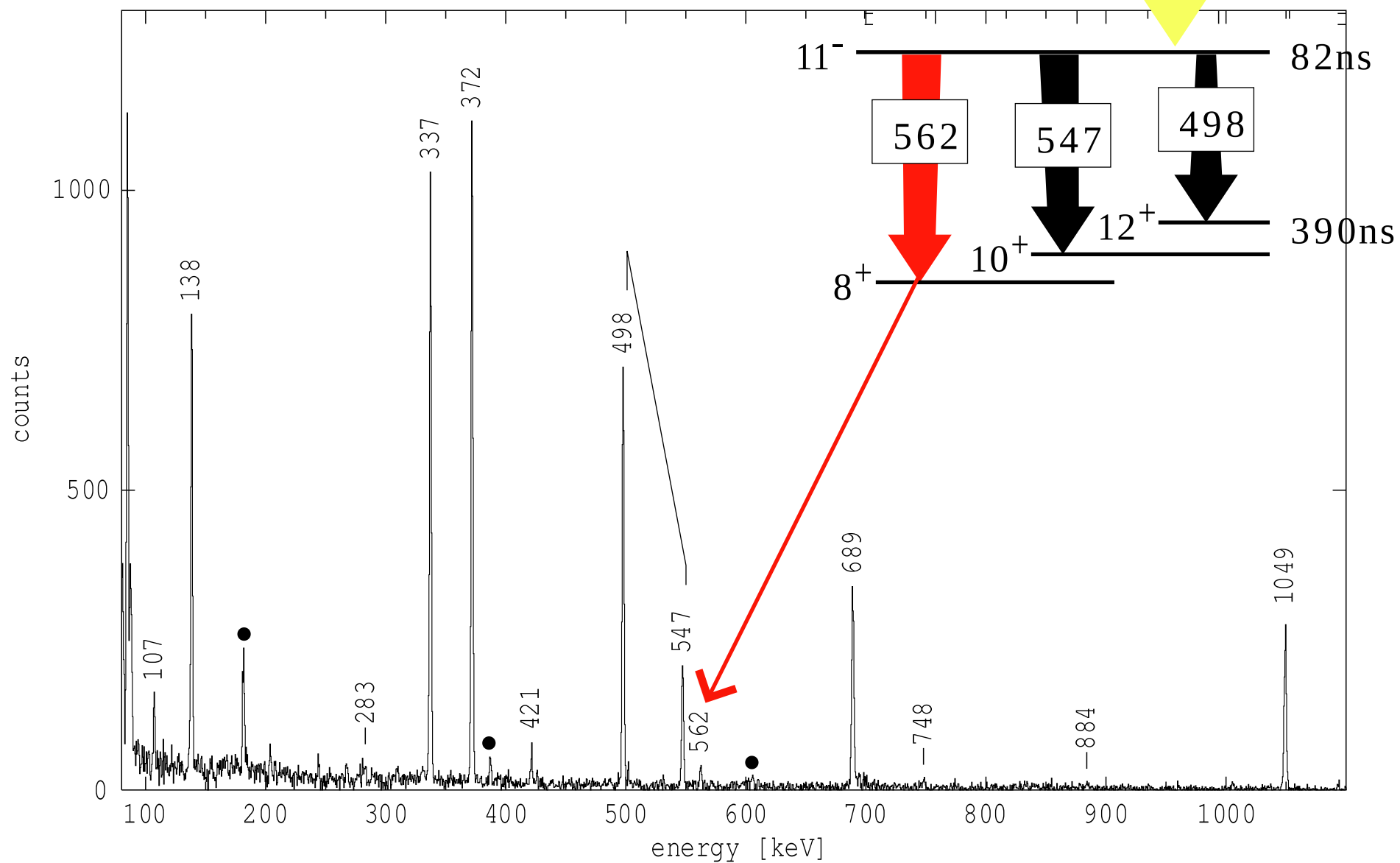


$I\gamma(498) / I\gamma(547) = \text{????}$
E3 Branches ????

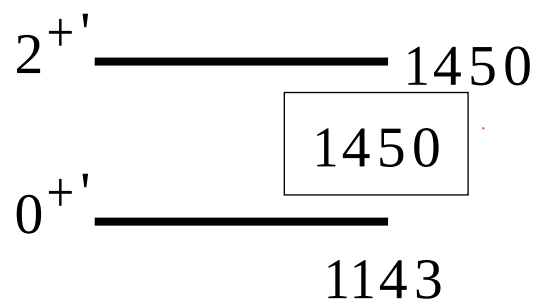
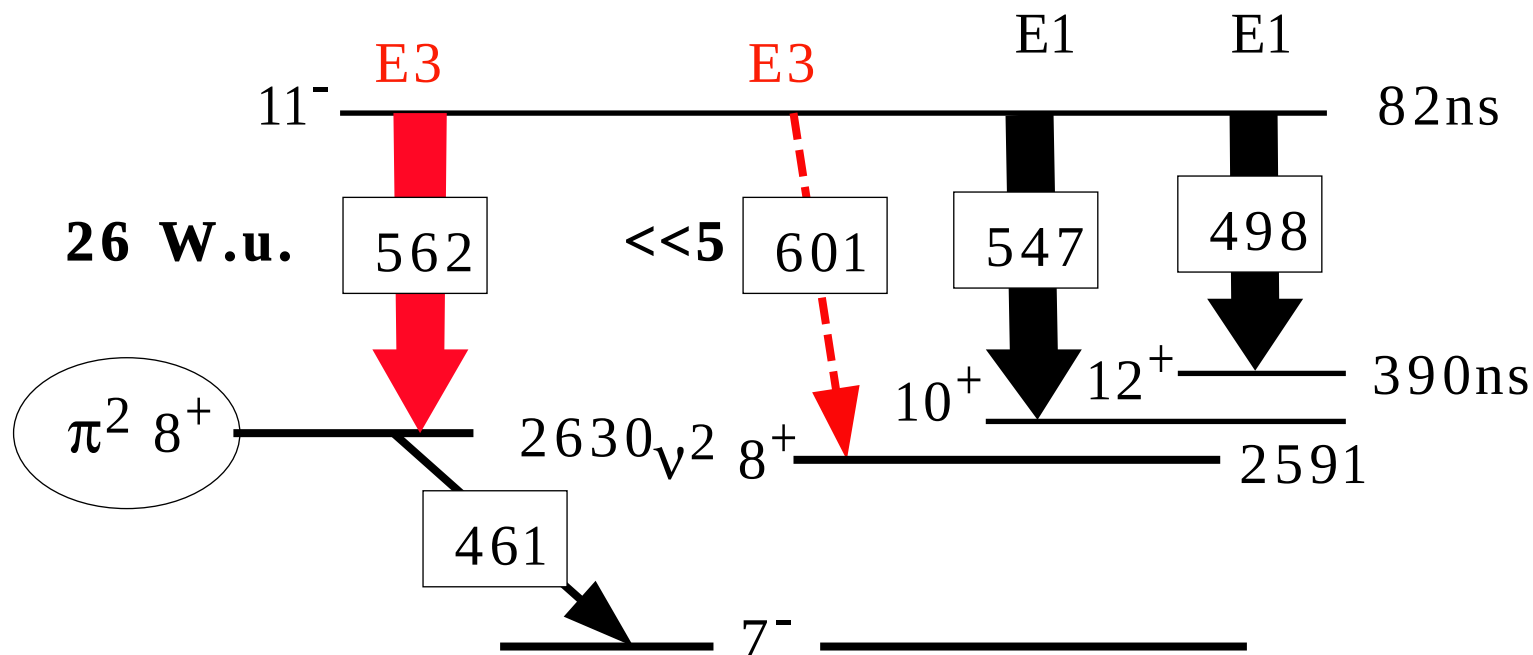
Penninga et al NPA 471(1987)535
Van Ruyven et al NPA 449(1986)
Singh et al NPA 707 (2002) 3

11^- Isomer decay ^{196}Pb

γ -ray gates above
plus time gate



11^- to 8^+ E3 ; ^{196}Pb solution ?



$$I\gamma(498) / I\gamma(547) = 3$$

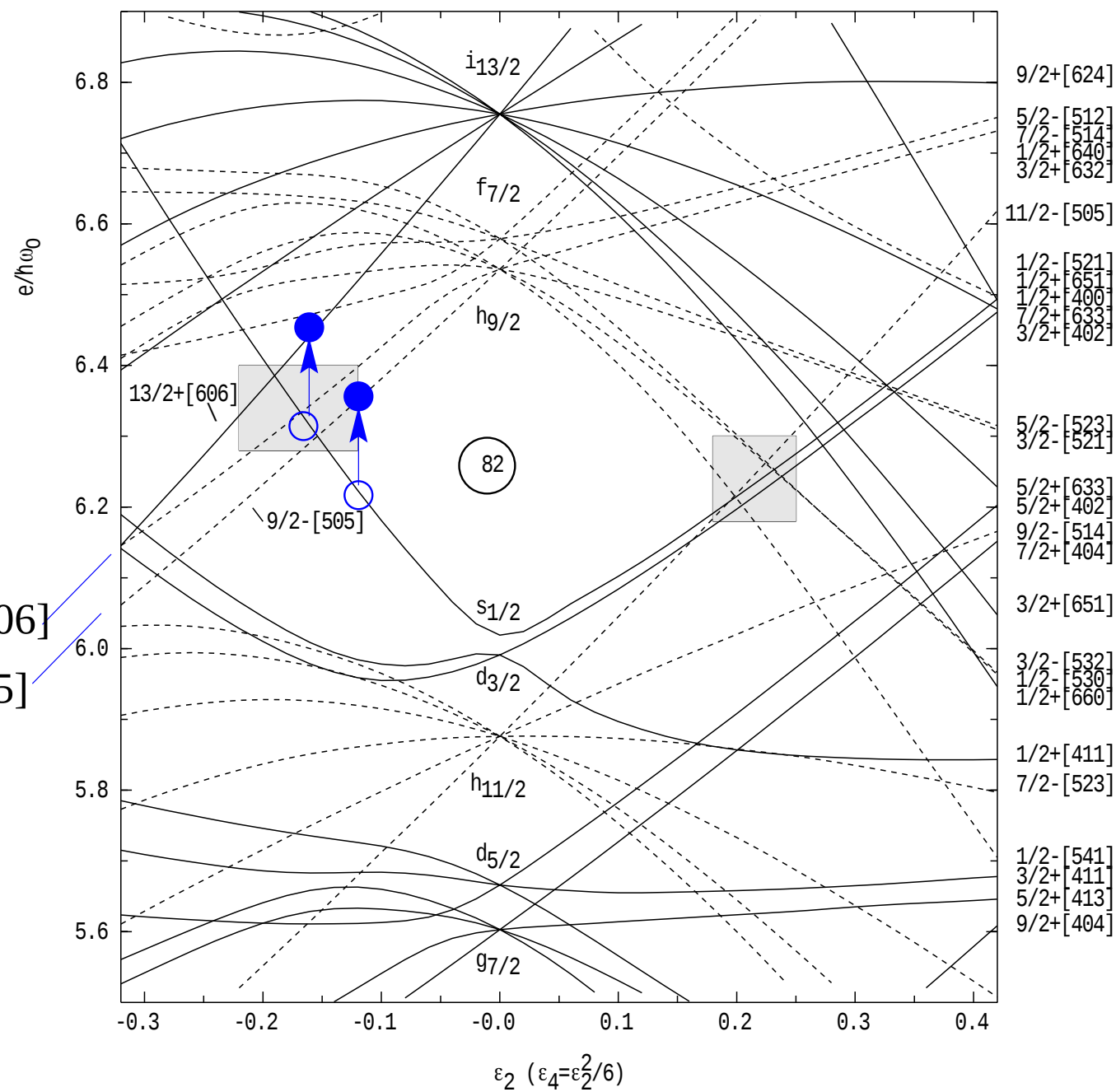
E3 Branch 562 keV

Type-A E3 ~ 20 W.u.

$K = 11-$ {

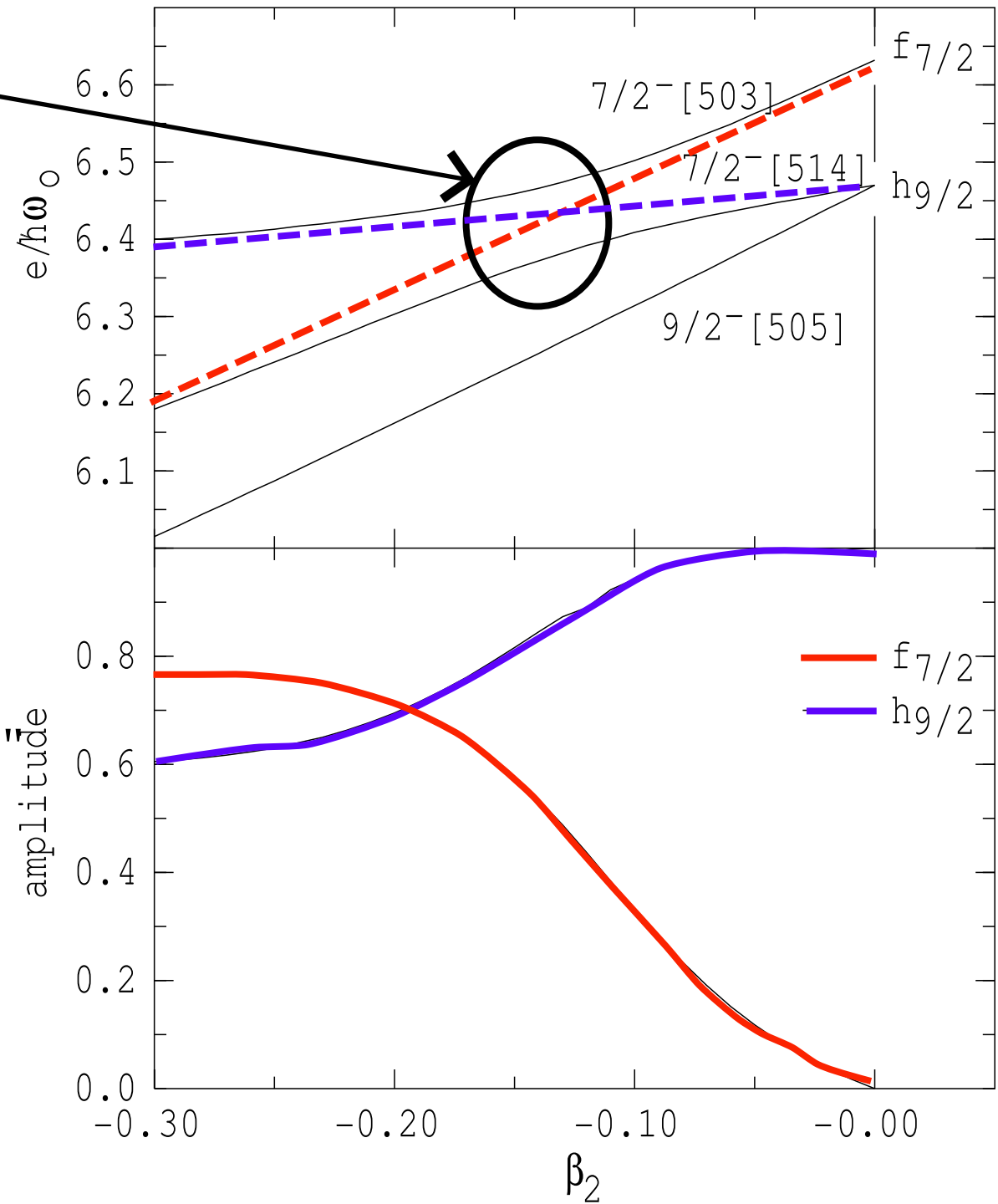
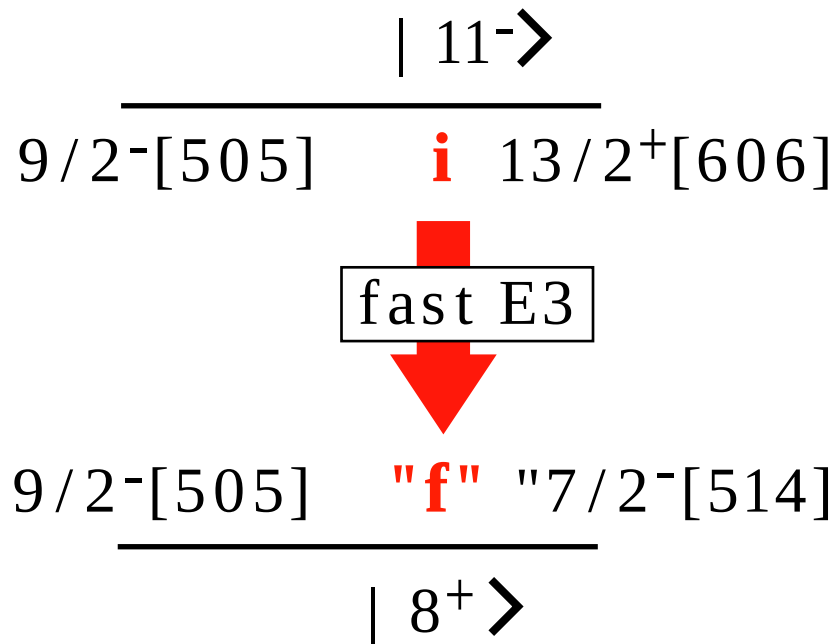
 $13/2+[606]$

 $9/2-[505]$



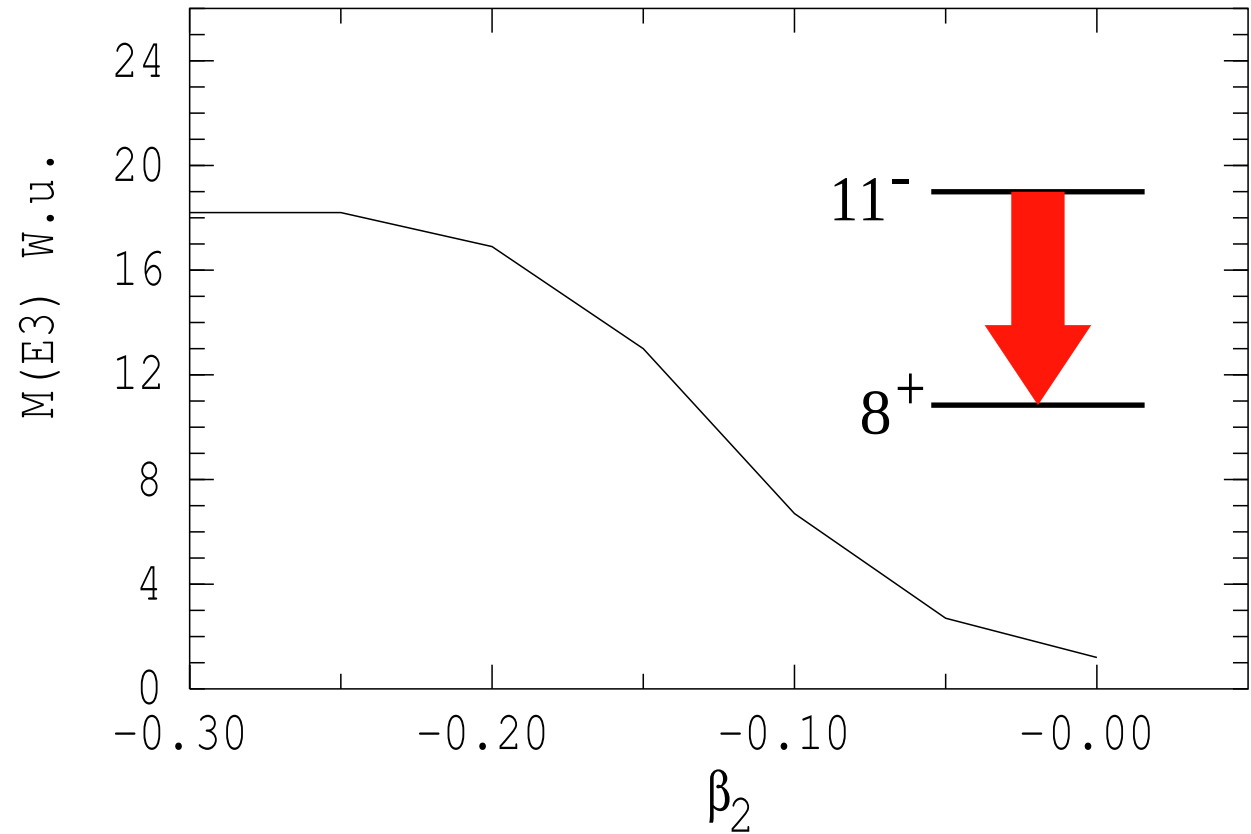
potential-mixing with oblate deformation

lowest $7/2^-$
proton orbital
changes from
 $h_{9/2}$ to $f_{7/2}$



approximate dependence on oblate deformation

Taking calculated amplitudes α and β of $h_{9/2}$ and $f_{7/2}$ configuration in "7/2-[514]" orbital in typical Nilsson Potential

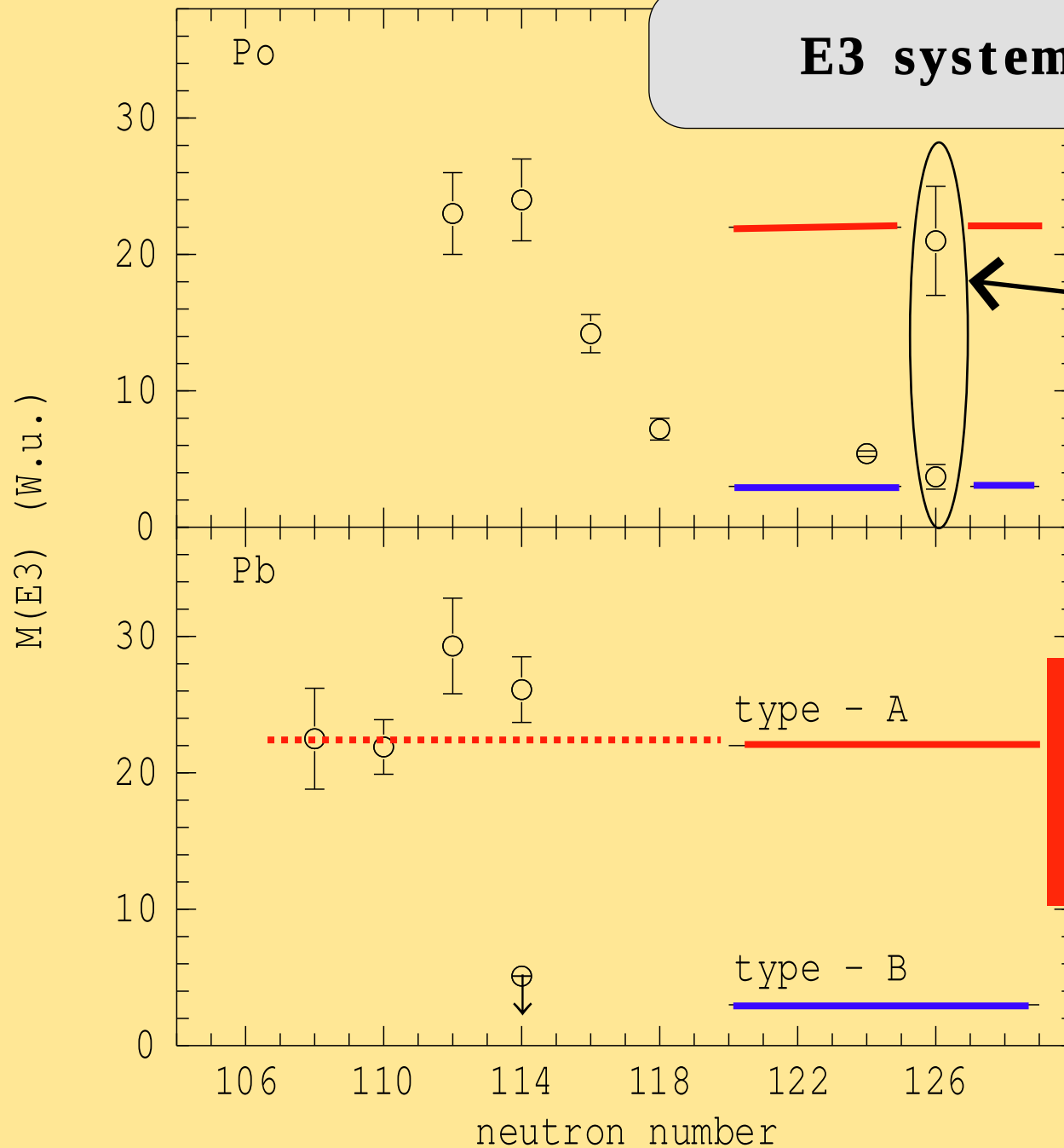


$B(E3, 11^- \text{ to } 8^+)$

$$\sim \{ \alpha U(\dots) \sqrt{B(E3; i_{13/2} \text{ to } h_{9/2})} + \beta U(\dots) \sqrt{B(E3; i_{13/2} \text{ to } f_{7/2})} \}^2$$

$$M(E3, 11^- \text{ to } 8^+) = \{ \alpha \, 0.632 \, \sqrt{3} + \beta \, 1.0 \, \sqrt{22} \}^2 \text{ W.u.}$$

E3 systematics ; Pb-Po

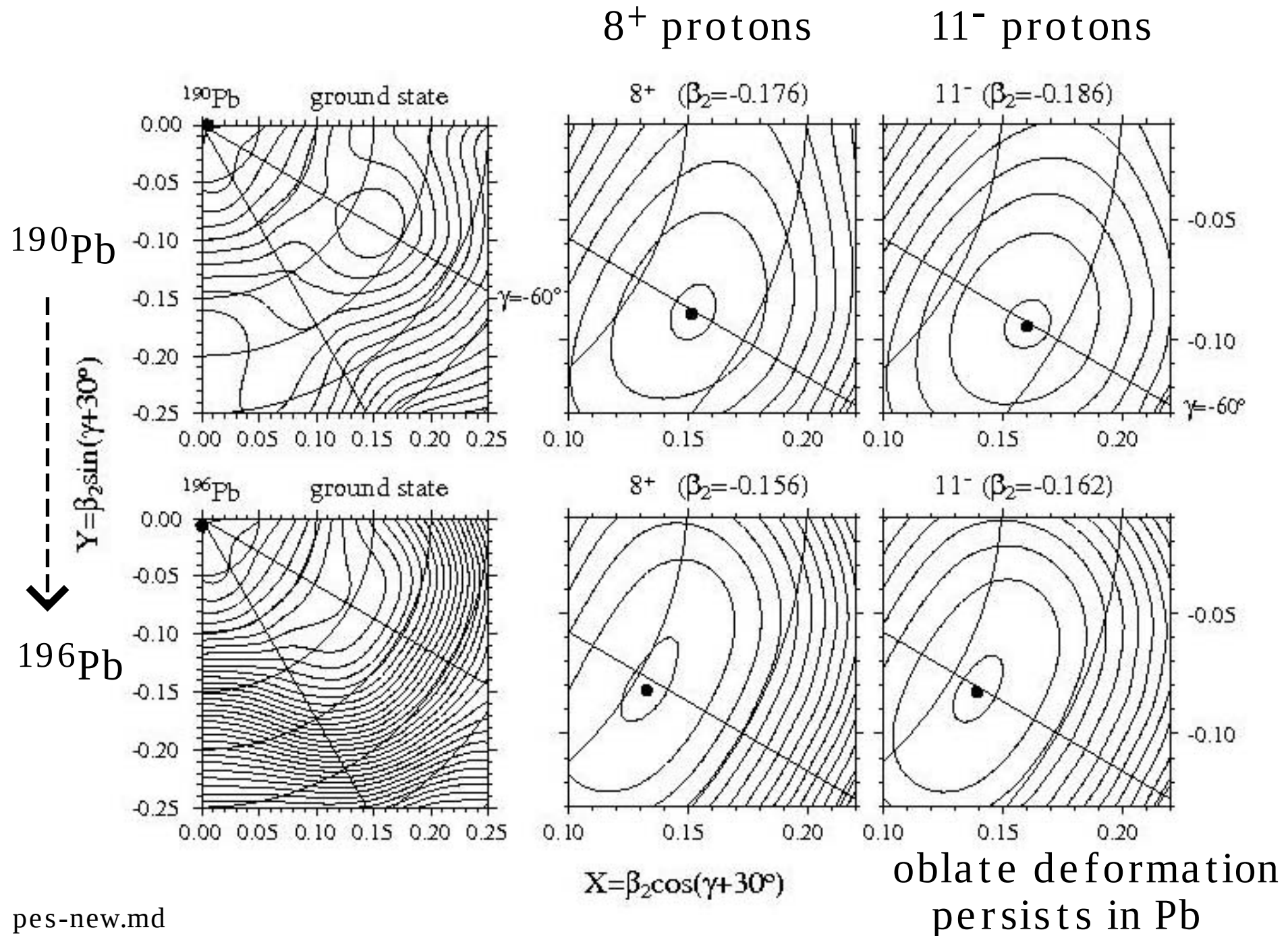


N=126 reproduced
by shell model with
octupole coupling
Stuchbery et al.
NPA 555 (1993)355

**Type - A E3 ~ 22 W.u.
consistent with ~constant
Oblate deformation
N < 116 ?**

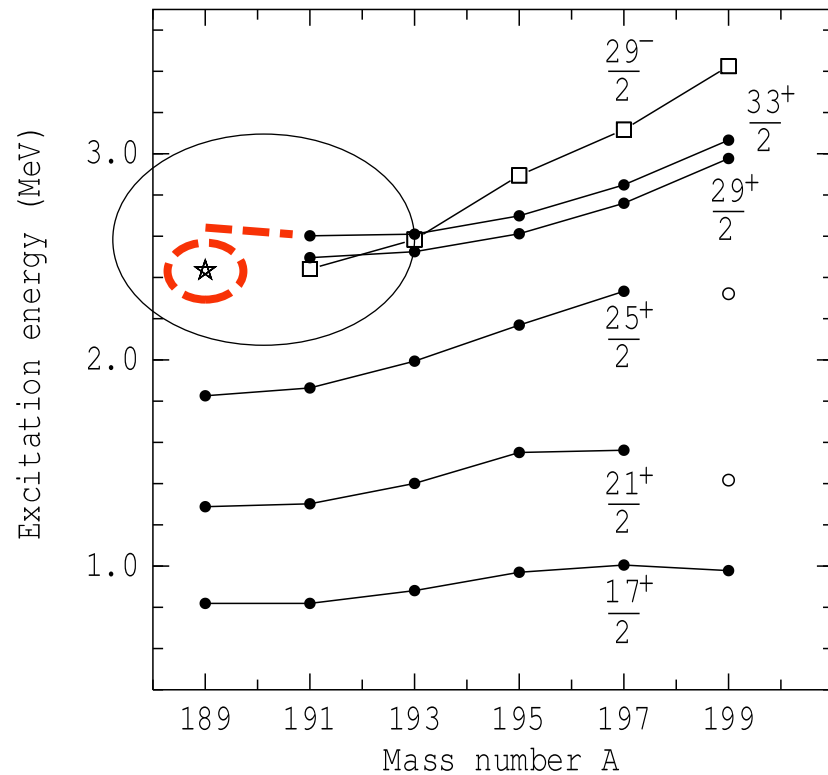
to be published...

PES - configuration constrained



CASE IV; ^{189}Pb isomer

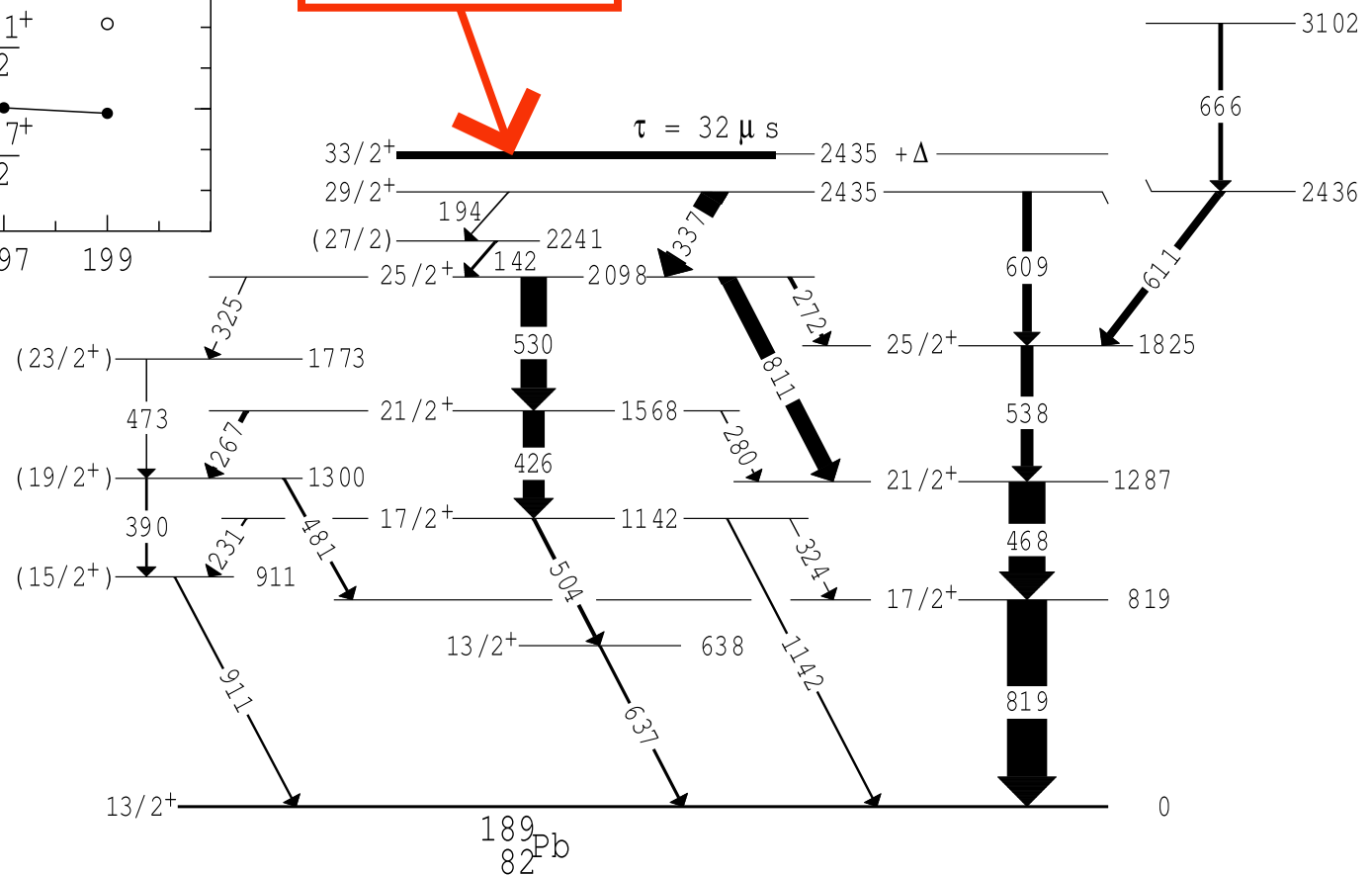
^{189}Pb : neutron $i_{13/2}^{-3}$



Baxter et al,
Phys Rev. C 71, 054302 (2005)

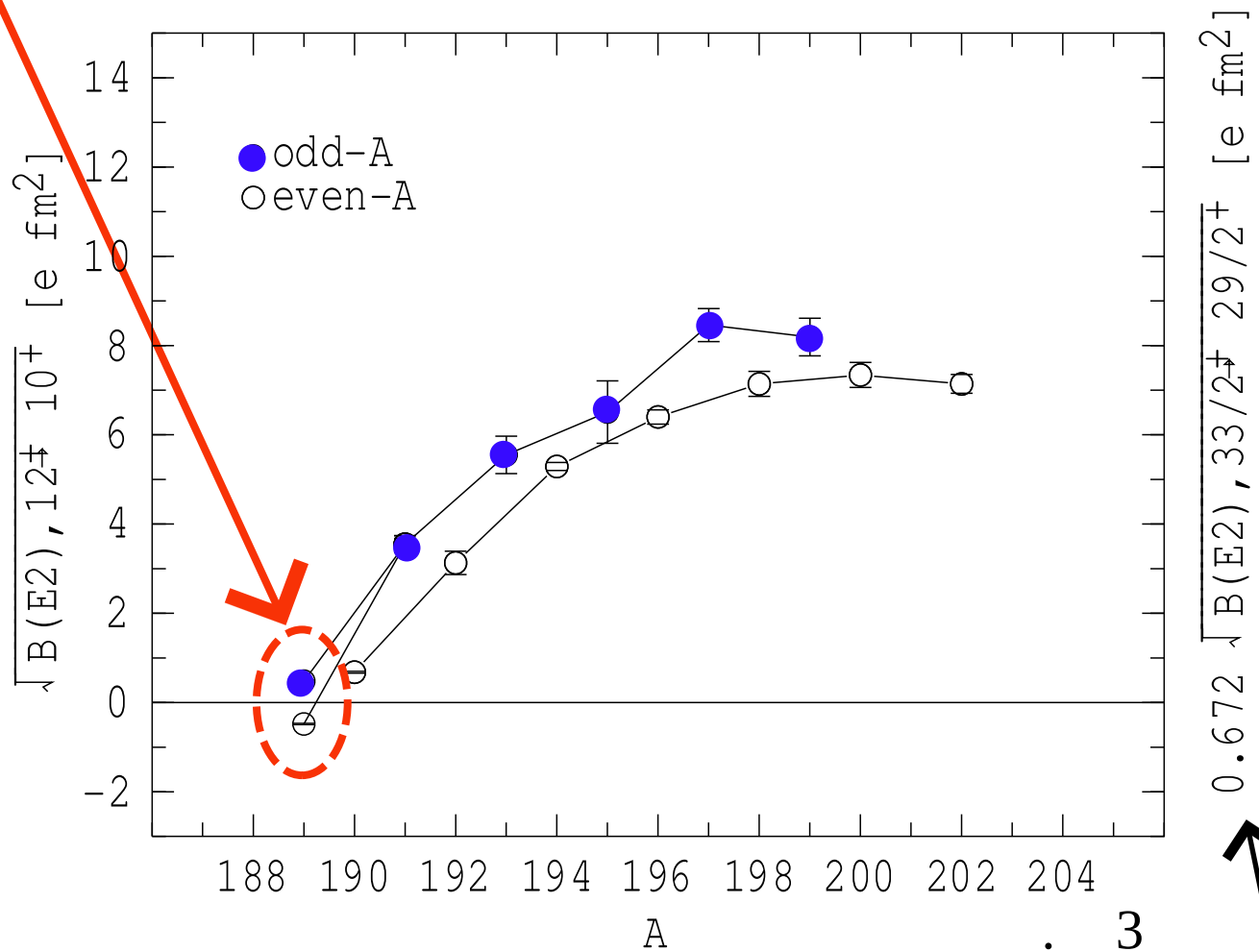
Tentative $J^\pi = 33/2^+$ assignment
with unobserved E2 transition

$\tau = 32 \mu\text{s}$



B(E2) mid-shell cancellation...

^{189}Pb proposed $33/2^+$ isomer

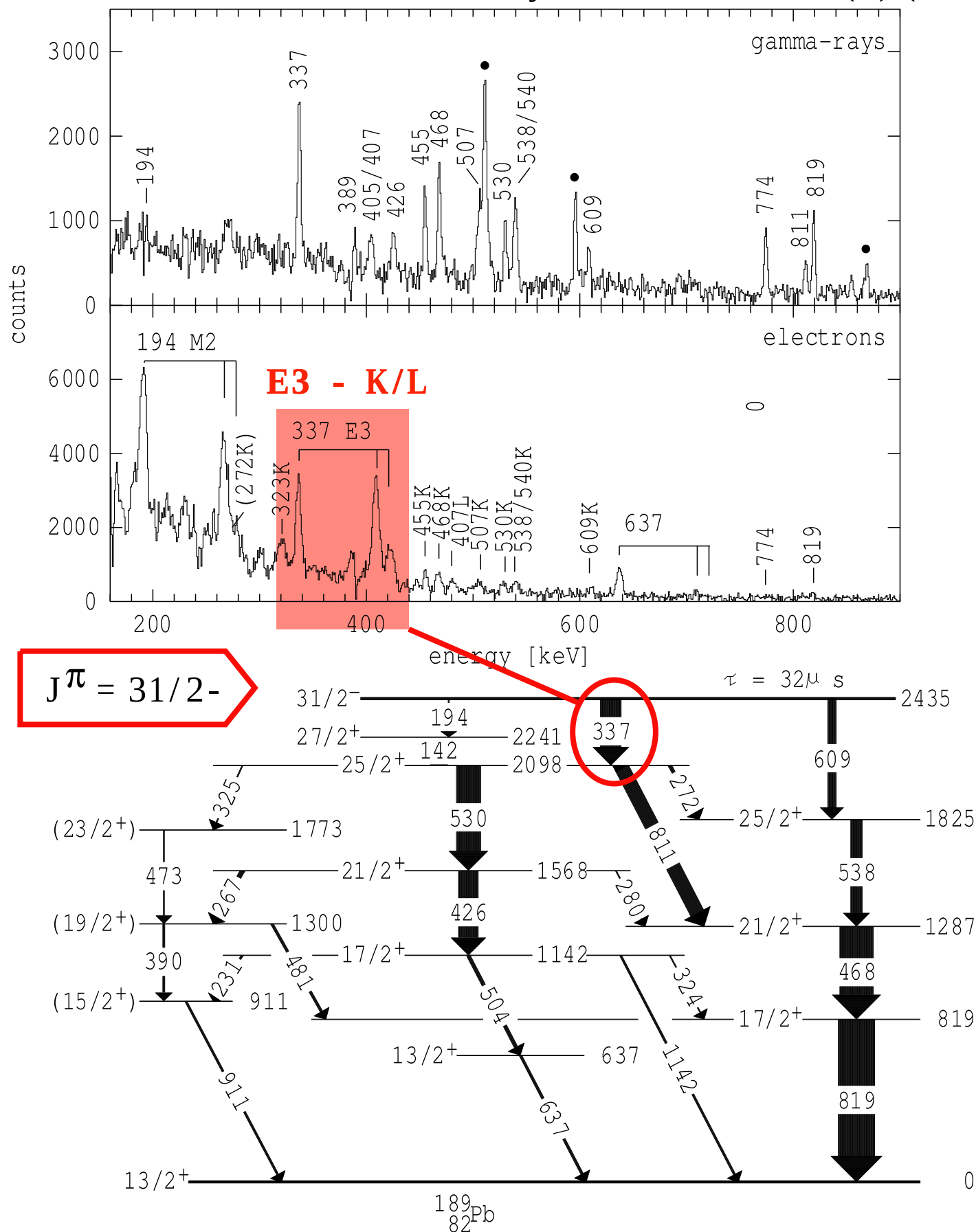


$i_{13/2}^3$ scaling to $i_{13/2}^2$

Solenogam test; ^{189}Pb 32 μs Isomer

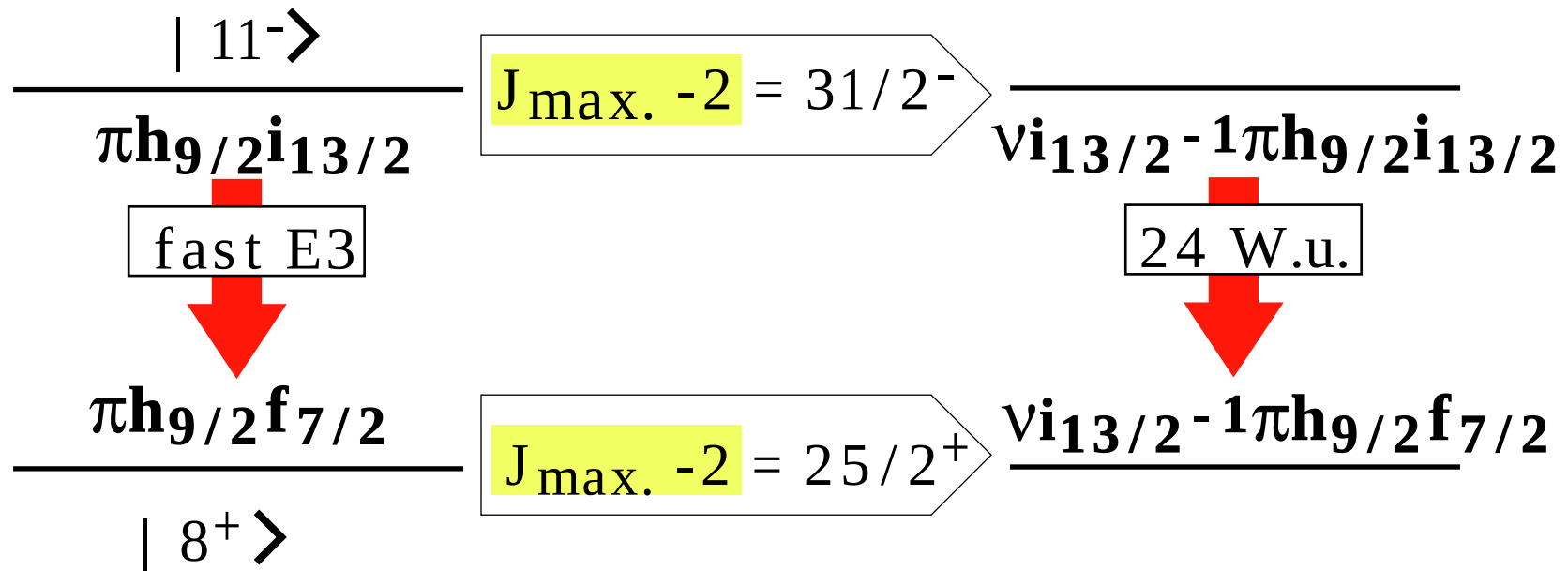
Characterisation as a Shears-mode bandhead

GDD, Lane, Kibedi. Nieminen. Phys Rev C 79,01302(R) (2009).



Configuration

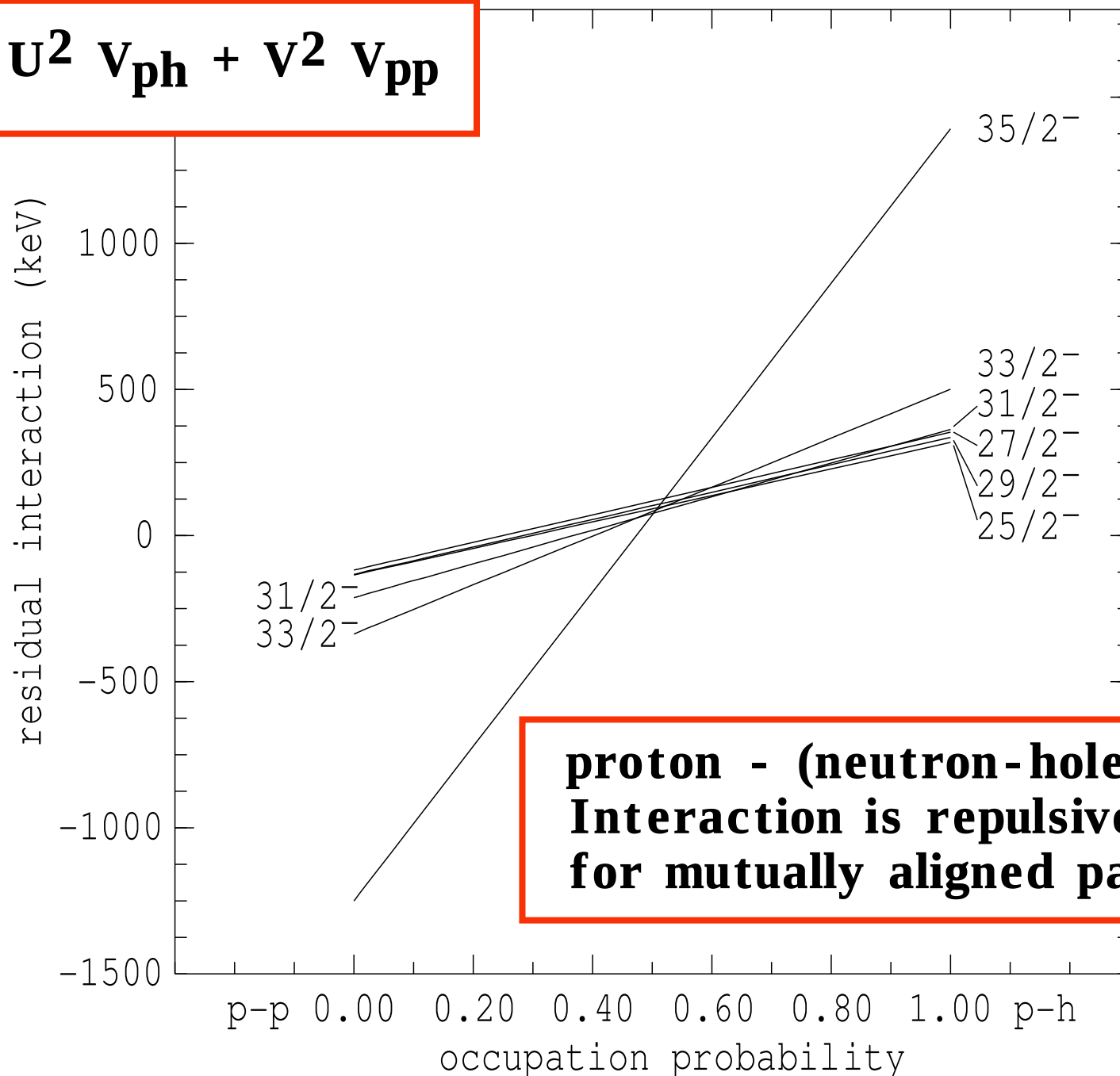
naive expectation:
Maximum coupling of
 11^- core and $i_{13/2}$ neutron hole $J^\pi = 35/2^-$



Non-Maximal Coupling -
balance of residual interactions

Residual Interactions: Particles/Holes

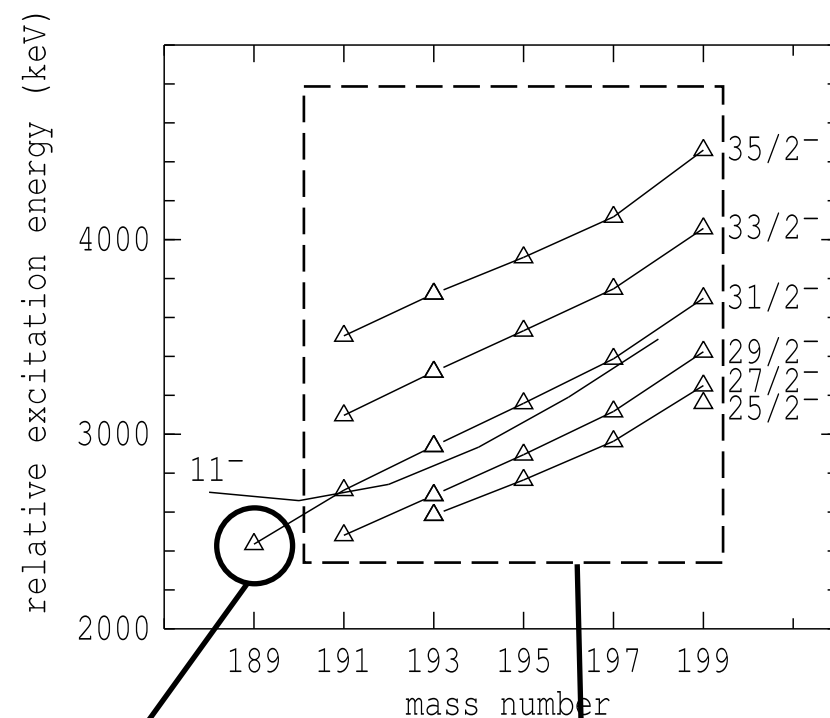
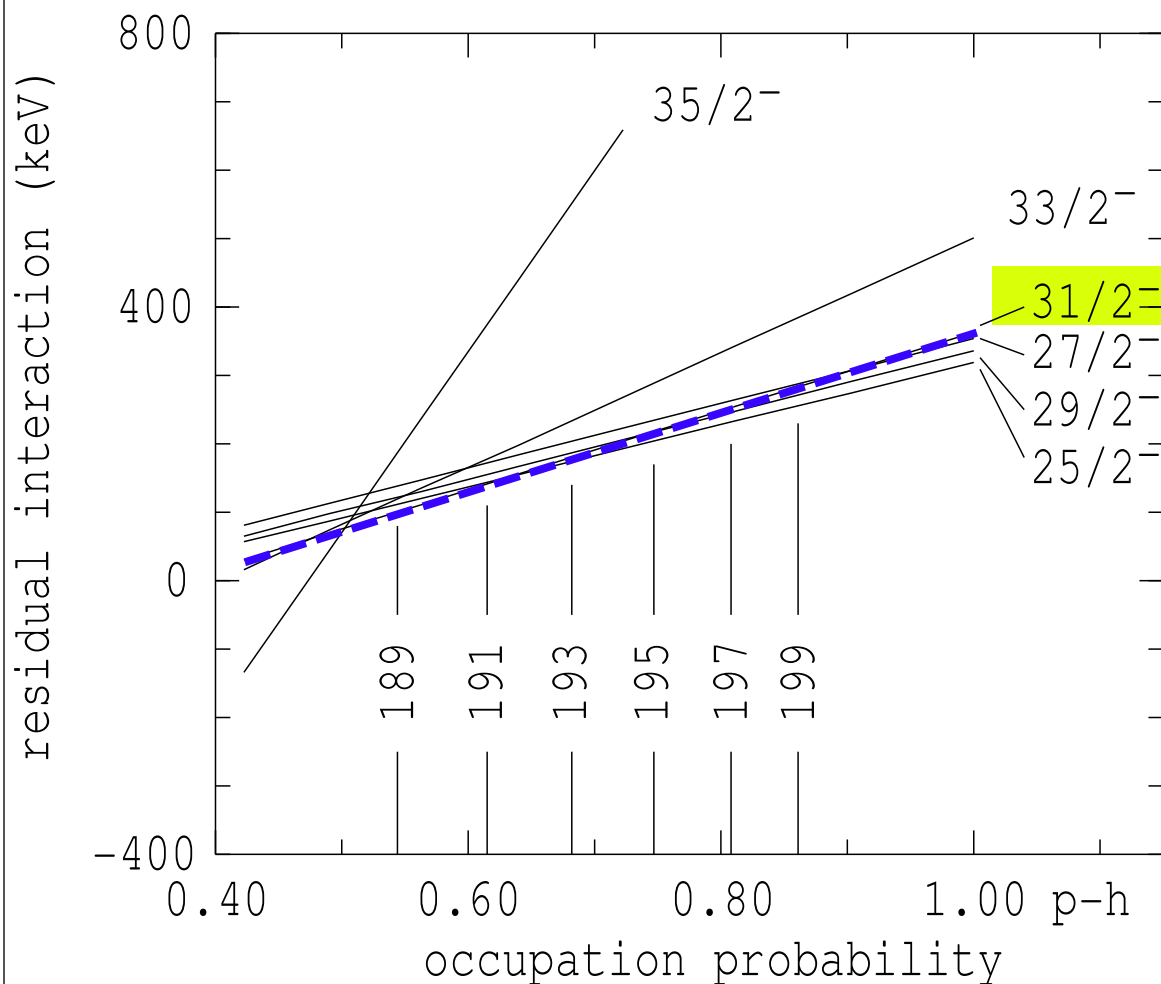
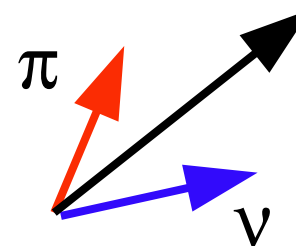
$$V_{\text{eff.}} = U^2 V_{\text{ph}} + V^2 V_{\text{pp}}$$



**proton - (neutron-hole)
Interaction is repulsive
for mutually aligned particles**

Residual Interactions: Detail/shears

$$\pi [h_{9/2}i_{13/2}]_{11} - \otimes \nu i_{13/2}$$



shears bands...

31/2⁻ Isomer

Final Comments...

**Isomers are a useful and important tool
in Nuclear Structure**

The detail matters

Expectation is not the same as Measurement

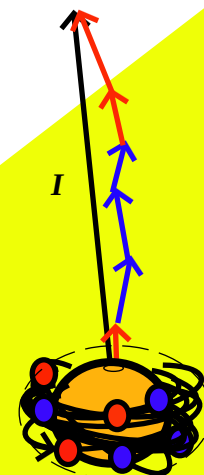
A Plea:

Don't ignore what has gone before

a thought for the day

Isomers have a life of their own

Dafni et al. PRL 55 (1985) 1271
 Quadrupole Moment.. $63/2^-$ isomer in ^{211}Rn
 "Absence of Core Deformations at Very High Spins"



$^{211}\text{Rn} (63/2^-) = ^{212}\text{Rn} (30^+) + \nu (3/2^-)$

Spherical (eff. charges)

$Q(\text{shell}) = -186$

$\Sigma_l Q_{sp}$

Single Particle
 Moments (e.f.m²)

$\pi h_{9/2}$ -43

$\pi i_{13/2}$ -62

$\nu f_{5/2}$ +20

$\nu g_{9/2}$ -29

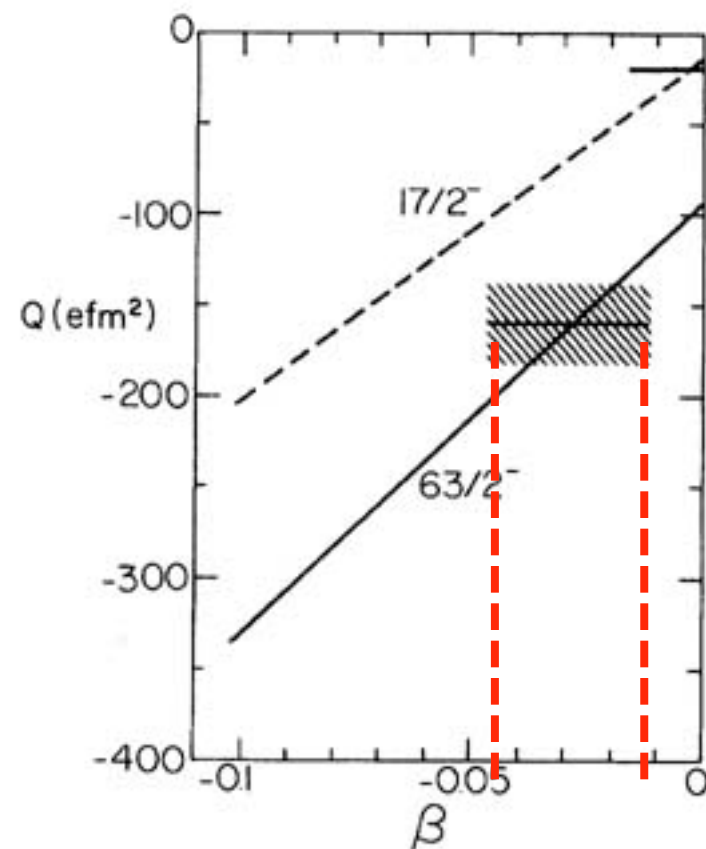
$\nu i_{11/2}$ -28

$\nu j_{15/2}$ -42

$|Q_{\text{exp}}| = 160(22)$

Deformed (no eff. charges)

X $Q = f(I)Q_0 = f(I)(Q_I + Q_{\text{core}})$
 $Q_{\text{core}} \propto Z_{\text{core}} R^2 (\beta + 0.36\beta^2)$



$\beta \sim 0.027(10)$

Also, see Sagawa&Arima PLB 202 (1988) 15