Three- quasiparticle-plus-rotor Coriolis coupling calculations

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PLAN OF TALK

- Motivation
- Three- quasiparticle plus axially symmetric rotor model (3QPRM)
- Choice of parameters and special issues
- Results
- Future improvement over present calculations

MOTIVATION

- Signature Splitting and Signature inversion-

  Total 49 bands exhibit Signature Effects

  (Singh *et al.* ADNDT 92 (2006)1)
- No quantitative calculations (for 3qp bands) using PPRM approach reported in literature
- Role of Coriolis and Particle-Particle interaction in Signature effects
- Phase of staggering within the members of given quadruplet

THREE-QUASIPARTICLE PLUS ROTOR MODEL

Salient features of The Model

- Present version of the TQPRM will work for all types (***nnn, ppp, npp, pnn***) of the 3qp bands
- Empirical values of residual interactions are considered without taking non- diagonal contributions
- 3qp bands which start out as the 3qp bands at band-head, leaving out those bands which start out as the 1qp bands and develops into the 3qp bands after band-crossing
- The vibrational interaction has been neglected

METHODOLGY

$$H = H_{\text{intrinsic}} + H_{\text{collective}}$$

$$H_{\text{intrinsic}} = H_{av} + H_{pair} + H_{res}$$

$$\Delta_p = \frac{1}{4}\{B(N,Z-2) - 3B(N,Z-1) + 3B(N,Z) - B(N,Z+1)\}$$

$$\Delta_n = \frac{1}{4}\{B(Z,N-2) - 3B(Z,N-1) + 3B(Z,N) - B(Z,N+1)\}$$

A. Bohr, B.R. Mottelson, Nuclear Structure: Volume 2
Audi and Wapstra, NPA 595 (1995)409

Residual Interaction

$$E_{res} = \sigma_{(1,2)} \left\{ \left[\frac{1}{2} - \delta_{\Sigma(1,2),0} \right] E^{split}_{(1,2)} - \delta_{K(1,2),0} E^{N}_{(1,2)} \Pi_{(1,2)} \right\} +$$

$$\sigma_{(2,3)} \left\{ \left[\frac{1}{2} - \delta_{\Sigma(2,3),0} \right] E^{split}_{(2,3)} - \delta_{K(2,3),0} E^{N}_{(2,3)} \Pi_{(2,3)} \right\} +$$

$$\sigma_{(1,3)} \left\{ \left[\frac{1}{2} - \delta_{\Sigma(1,3),0} \right] E^{split}_{(1,3)} - \delta_{K(1,3),0} E^{N}_{(1,3)} \Pi_{(1,3)} \right\}$$

Jain & Jain PRC 45 (1992)3013

$$H_{collective} = H_{rotational} + H_{vibrational}$$

$$H = H_{intrinsic} + H_{rotational}$$

$$H_{rotational} = H_{rot}^{o} + H_{irrot} + H_{ppc} + H_{rpc}$$

$$H_{rot}^{o} = \frac{\hbar^2}{2\Im}\left[I^2 - I_z^2 \right]$$

Rotational contribution

$$H_{irrot} = \frac{\hbar^2}{2\Im}\left[(j_1^2 - j_{1z}^2) + (j_2^2 - j_{2z}^2) + (j_3^2 - j_{3z}^2) \right]$$

Intrinsic contribution

$$H_{ppc} = \frac{\hbar^2}{2\Im}\left[(j_{1+} j_{2-} + j_{1-} j_{2+}) + (j_{2+} j_{3-} + j_{2-} j_{3+}) + (j_{1+} j_{3-} + j_{1-} j_{3+}) \right]$$

particle-particle interaction

$$H_{rpc} = -\frac{\hbar^2}{2\Im}\left[I_+ J_- + I_- J_+ \right]$$

rotor-particle interaction

$$H = H_{intrinsic} + H_{rotational}$$

$$|IMK\alpha\rangle = \sqrt{\frac{2I+1}{16\pi^2}}\left[|D^I_{MK}\rangle |K\alpha\rangle + (-1)^{I+K} |D^I_{M-K}\rangle R_x(\pi) |K\alpha\rangle \right]$$

CALCULATION OF THE MATRIX ELEMENTS

$$\langle IMK'\alpha' | H^o_{rot} | IMK\alpha\rangle = \langle IMK'\alpha' | \frac{\hbar^2}{2\Im}\left[I^2 - I_z^2 \right] | IMK\alpha\rangle =$$

$$= \frac{\hbar^2}{2\Im}\left[I(I+1) - K^2 \right] \delta_{K'K}\delta_{\alpha'\alpha}$$

$$\langle IMK'\alpha' | H_{irrot} | IMK\alpha \rangle = \langle IMK'\alpha' | \frac{\hbar^2}{2\Im}\left[(j_1^2 - j_{1z}^2) + (j_2^2 - j_{2z}^2) + (j_3^2 - j_{3z}^2)\right] | IMK\alpha \rangle$$

$$= \frac{\hbar^2}{2\Im}\left[\begin{array}{l}\left(\sum_{j_1}\left|C_{k_1}^{j_1}\right|^2 j_1(j_1+1) - k_1^2\right) + \left(\sum_{j_2}\left|C_{k_2}^{j_2}\right|^2 j_2(j_2+1) - k_2^2\right) \\ + \left(\sum_{j_3}\left|C_{k_3}^{j_3}\right|^2 j_3(j_3+1) - k_3^2\right)\end{array}\right]\delta_{K'K}\delta_{\alpha'\alpha}$$

Jain et al. RMP 62(1990)393, Moller et al, ADNDT 59(1995)185

$$\langle IMK'\alpha' | H_{ppc} | IMK\alpha \rangle = \langle IMK'\alpha' | \frac{\hbar^2}{2\Im}\left[\begin{array}{r}(j_{1^+} j_{2^-} + j_{1^-} j_{2^+}) + (j_{2^+} j_{3^-} + j_{2^-} j_{3^+}) \\ + (j_{1^+} j_{3^-} + j_{1^-} j_{3^+})\end{array}\right] | IMK\alpha \rangle$$

$$=\frac{\hbar^2}{2\Im}\begin{bmatrix}\left(\langle k_1'\rho_1'|\,j_{1^+}|k_1\rho_1\rangle\langle k_2'\rho_2'|\,j_{2^-}|k_2\rho_2\rangle+\langle k_1'\rho_1'|\,j_{1^-}|k_1\rho_1\rangle\langle k_2'\rho_2'|\,j_{2^+}|k_2\rho_2\rangle\right)\delta_{k_3'k_3}\delta_{\rho_3'\rho_3}\\ +\left(\langle k_2'\rho_2'|\,j_{2^+}|k_2\rangle\langle k_3'\rho_3'|\,j_{3^-}|k_3\rho_3\rangle+\langle k_2'\rho_2'|\,j_{2^-}|k_2\rho_2\rangle\langle k_3'\rho_3'|\,j_{3^+}|k_3\rho_3\rangle\right)\delta_{k_1'k_1}\delta_{\rho_1'\rho_1}\\ +\left(\langle k_1'\rho_1'|\,j_{1^+}|k_1\rho_1\rangle\langle k_3'\rho_3'|\,j_{3^-}|k_3\rho_3\rangle+\langle k_1'\rho_1'|\,j_{1^-}|k_1\rho_1\rangle\langle k_3'\rho_3'|\,j_{3^+}|k_3\rho_3\rangle\right)\delta_{k_2'k_2}\delta_{\rho_2'\rho_2}\end{bmatrix}\delta_{K'K}$$

$$\langle IMK'\alpha'|H_{rpc}|IMK\alpha\rangle=\langle IMK'\alpha'|-\frac{\hbar^2}{2\Im}\left[I_+J_-+I_-J_+\right]|IMK\alpha\rangle$$

$$=-\frac{\hbar^2}{2\Im}\begin{bmatrix}\left(\sqrt{(I+K)(I-K+1)}\langle k_1'\rho_1'k_2'\rho_2'k_3'\rho_3'|\,j_{1^-}+j_{2^-}+j_{3^-}|k_1\rho_1k_2\rho_2k_3\rho_3\rangle\right)\delta_{K',K-1}\\ +\left(\sqrt{(I-K)(I+K+1)}\langle k_1'\rho_1'k_2'\rho_2'k_3'\rho_3'|\,j_{1^+}+j_{2^+}+j_{3^+}|k_1\rho_1k_2\rho_2k_3\rho_3\rangle\right)\delta_{K',K+1}\\ +(-1)^{I+K}\left(\sqrt{(I+K)(I-K+1)}\langle k_1'\rho_1'k_2'\rho_2'k_3'\rho_3'|\left(j_{1^+}+j_{2^+}+j_{3^+}\right)R_x|k_1\rho_1k_2\rho_2k_3\rho_3\rangle\right)\delta_{K',-K+1}\end{bmatrix}$$

VARIOUS CASES OF COUPLINGS

$$(A1)\quad K=\left|k_1+k_2+k_3\right|,\quad K'=\left|k_1'+k_2'+k_3'\right|$$

$$(B1)\quad K=\left|k_1+k_2+k_3\right|,\quad K'=\left|k_1'+k_2'-k_3'\right|$$

$$(B2)\quad K=\left|k_1+k_2+k_3\right|,\quad K'=\left|k_1'-k_2'+k_3'\right|$$

$$(B3)\quad K=\left|k_1+k_2+k_3\right|,\quad K'=\left|-k_1'+k_2'+k_3'\right|$$

$$(C1)\quad K=\left|k_1+k_2-k_3\right|,\quad K'=\left|k_1'+k_2'+k_3'\right|$$

$$(C2)\quad K=\left|k_1-k_2+k_3\right|,\quad K'=\left|k_1'+k_2'+k_3'\right|$$

$$(C3)\quad K=\left|-k_1+k_2+k_3\right|,\quad K'=\left|k_1'+k_2'+k_3'\right|$$

$$(D1)\quad K=\left|k_1+k_2-k_3\right|,\quad K'=\left|k_1'+k_2'-k_3'\right|$$

$$(D2)\quad K=\left|k_1+k_2-k_3\right|,\quad K'=\left|k_1'-k_2'+k_3'\right|$$

$$(D3)\quad K=\left|k_1+k_2-k_3\right|,\quad K'=\left|-k_1'+k_2'+k_3'\right|$$

$$(D4)\quad K=\left|k_1-k_2+k_3\right|,\qquad K'=\left|k_1'+k_2'-k_3'\right|$$

$$(D5)\quad K=\left|k_1-k_2+k_3\right|,\qquad K'=\left|k_1'-k_2'+k_3'\right|$$

$$(D6)\quad K=\left|k_1-k_2+k_3\right|,\qquad K'=\left|-k_1'+k_2'+k_3'\right|$$

$$(D7)\quad K=\left|-k_1+k_2+k_3\right|,\qquad K'=\left|k_1'+k_2'-k_3'\right|$$

$$(D8)\quad K=\left|-k_1+k_2+k_3\right|,\qquad K'=\left|k_1'-k_2'+k_3'\right|$$

$$(D9)\quad K=\left|-k_1+k_2+k_3\right|,\qquad K'=\left|-k_1'+k_2'+k_3'\right|$$

The matrix elements for different terms of Total Hamiltonian are calculated using above 16 coupling conditions. This leads to complexity of 3qp Coriolis mixing calculations.

CHOICE OF THE PARAMETERS AND MAJOR ISSUES INVOLVED

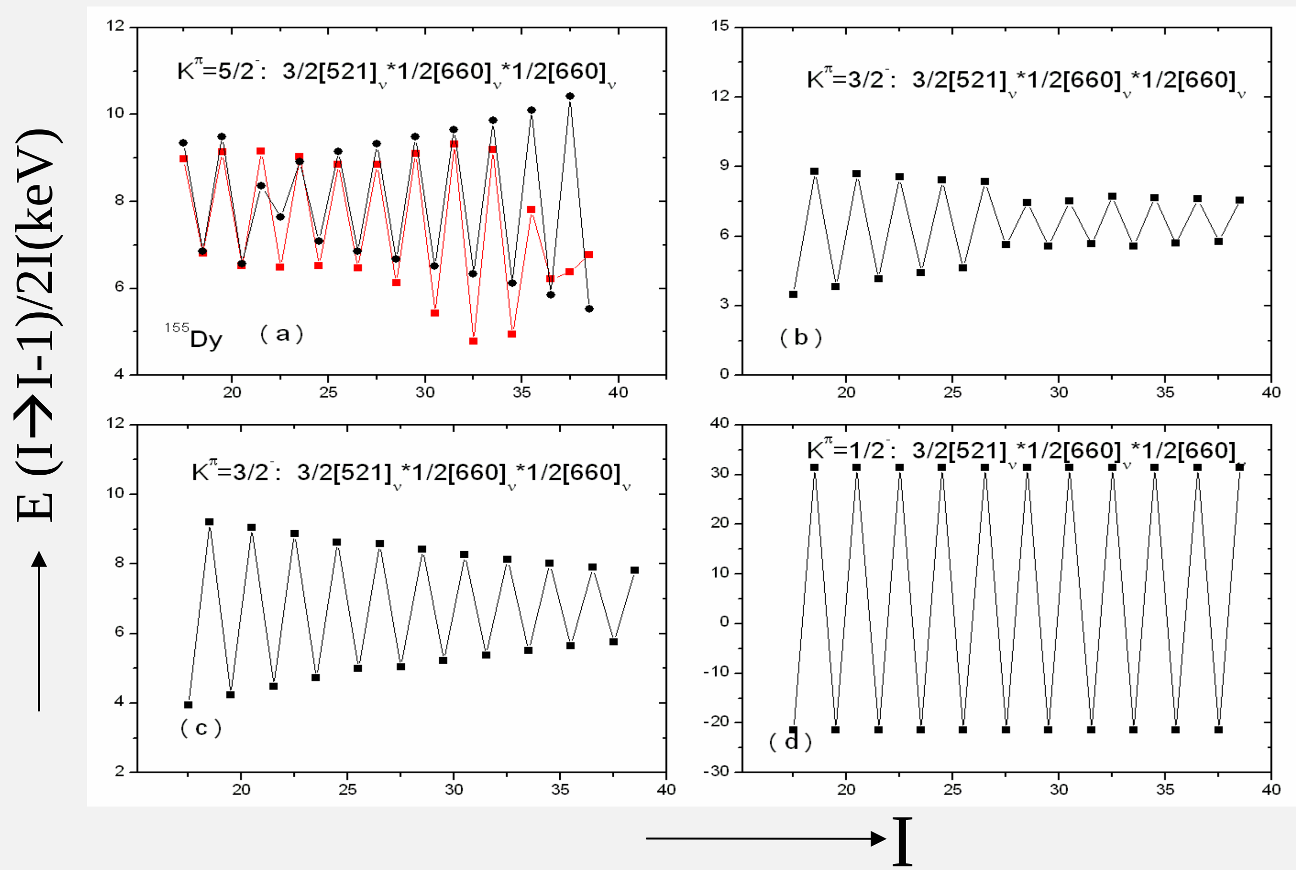
- For a given 3qp configuration, we have four band-heads and hence four different rotational bands which leads to complexity of Coriolis mixing calculations. In order to avoid this problem during the preliminary testing of the present model, we have considered only those 3qp configurations which involve a relatively low-Ω orbitals, so that we get a small basis and hence a relatively small number of interacting bands

- Estimation of appropriate value of inertia parameter for a given 3qp band. For experimentally observed 3qp bands, inertia parameter is calculated by using the first two energy levels and for other bands; it is taken as a free parameter.

- Most of the important bands taking part in the Coriolis mixing are not known. In order to overcome this problem we calculate the band-head energies for all the interacting bands. But the exact estimation of the band-head energies by using this model is still a problem due to non-availability of the experimental data for the Gallagher Moszkowski (GM) splitting as well as the Newby shift energies of all the two-quasiparticle (2qp) doublets comprising the 3qp configuration.

RESULTS AND DISCUSSION

Signature splitting in ^{155}Dy

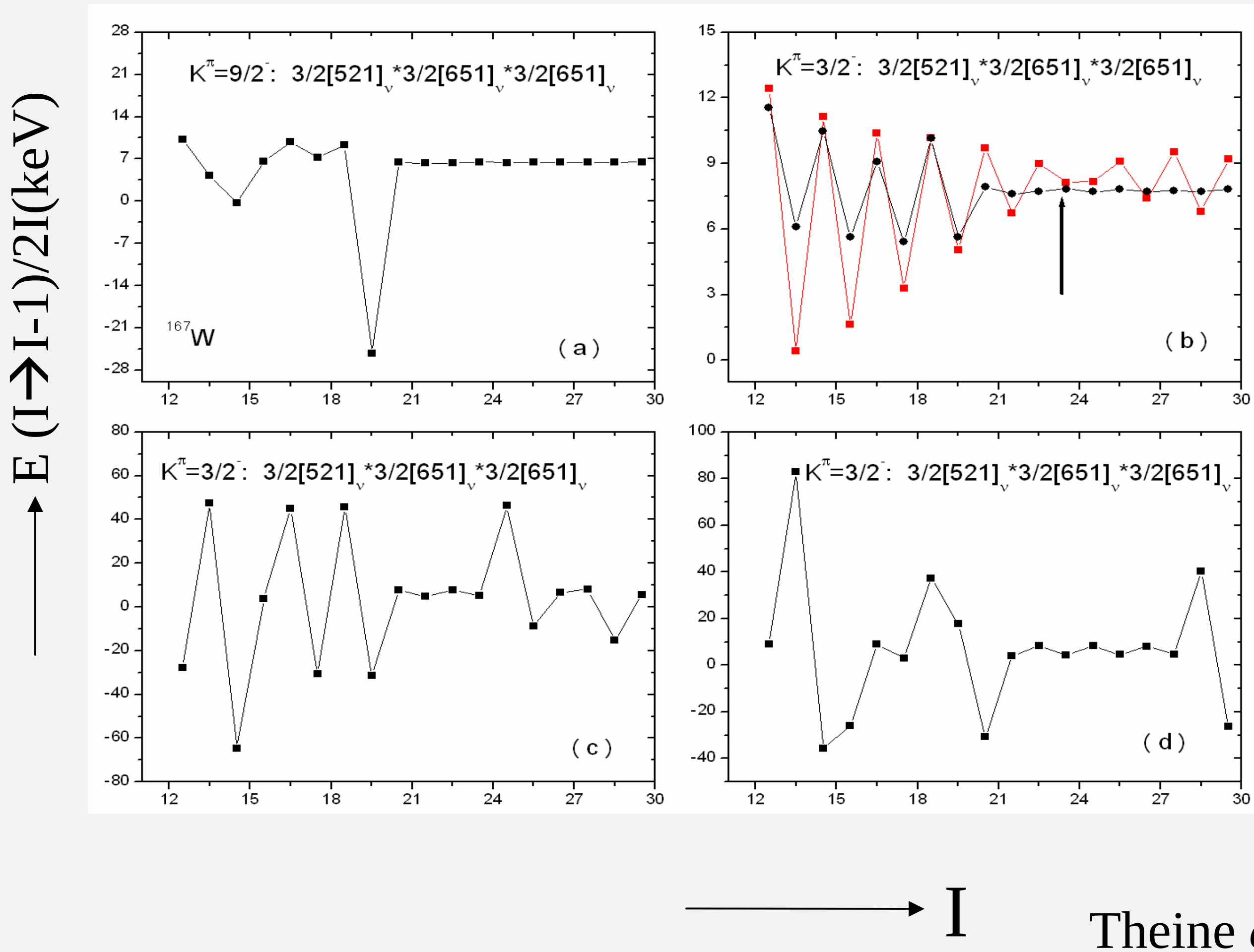


- Interested spin range is I=25/2 to 71/2
- Mag. and Phase confirms the validity of Model
- Spin assignments
- Phase of Staggering
- Higher Order Coriolis plays major role

Vlastou et al. NPA 580(1994)133

Basis of 48 bands; only one is experimentally observed !

Signature splitting in ^{167}W



- Phase, magnitude as well as point of inv-ersion is exactly reproduced
- confirms the validity of our model
- Spin assignments
- First Order Coriolis coupling plays major role

Theine *et al.* NPA 548(1992)71

Basis of 108 bands; only one is experimentally observed !

Attenuation of the $\boldsymbol{i}_{13/2}$ matrix elements by **45-50%** and the other matrix elements by **5-10%.** Only a minor variation in the inertia parameter has been done for the 3qp bands having considerable contribution to the given band

FUTURE IMPROVEMENTS

Functional minimization of input variables has not been done, which will improve comparison between calculations and experimental results.

(James and Roos, Comp. Phys Comm.10(1975)343)

COLLABORATORS

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- Prof. P.M. Walker, University of Surrey, UK
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